

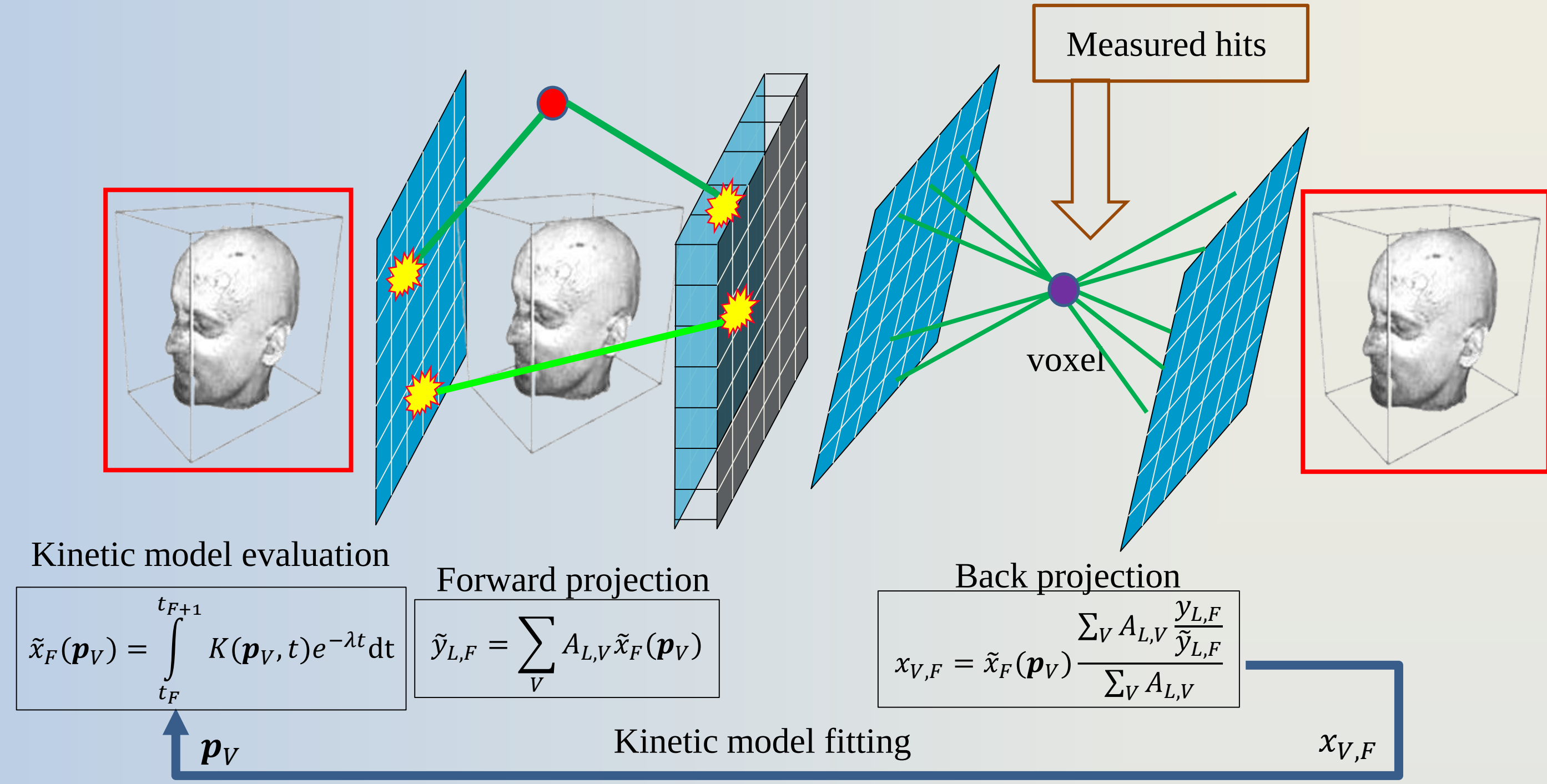
# Model-based Motion Compensation for PET

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**Abstract**—The self-motion of the examined patient often cannot be avoided during PET scan, thus the motion information should be measured and incorporated into the reconstruction process. Event-based approaches suffer from the missing data problem, i.e. not measured events cannot be transformed into the field of view, and are limited to rigid body motion. To allow the assumption that the system matrix is static in a frame, we would need very high number of frames when the motion is fast. To attack these problem, we re-formulate the ML-EM scheme to incorporate continuous motion. We show that the generalized ML-EM scheme should execute a convolution with the motion before forward projection, and two convolutions for the numerator and denominator of the back projection formula before division is executed. For the efficient GPU implementation of the motion convolution, we apply motion blur algorithms of computer graphics where the paths of object-space voxels are approximated by anti-aliasing line drawing methods.

## Dynamic ML-EM reconstruction WITHOUT motion compensation



## Dynamic ML-EM reconstruction WITH motion compensation

We distinguish two voxel arrays, an object-space one that is indexed with  $V$  and a tomograph-space one that is indexed with  $W$ . The object-space voxel array is fixed to the object, tomograph-space array is fixed to the tomograph.

### Idea of compensation:

Let us generalize the maximum-likelihood estimators for time dependent system matrix and approximate integrals of products by products of integrals.

### The reconstruction algorithm:

- For each frame, we compute the activity  $x_{V,F}$  of voxel  $V$  in frame  $F$ :

Evaluation of the integral of the radiotracer for each object-space voxel  $V$  and for this frame  $F$ :

$$\tilde{x}_F(\mathbf{p}_V) = \int_{t_F}^{t_{F+1}} K(\mathbf{p}_V, t) e^{-\lambda t} dt$$

- Motion blur with overlap factor  $o_{V,W,F}$  to compute the activity of tomograph-space voxels  $W$ :

$$\tilde{x}_{W,F} = \sum_V \tilde{x}_F(\mathbf{p}_V) o_{V,W,F}$$

- Forward projection of the tomograph-space voxels to obtain expected LOR hits  $\tilde{y}_{L,F}$

$$\tilde{y}_{L,F} = \sum_W A_{L,W} \tilde{x}_{W,F}$$

- Computation of numerator and denominator of the back projection

$$N_{W,F} = \sum_W A_{L,W} \frac{y_{L,F}}{\tilde{y}_{L,F}} \quad D_{W,F} = \sum_W A_{L,W}$$

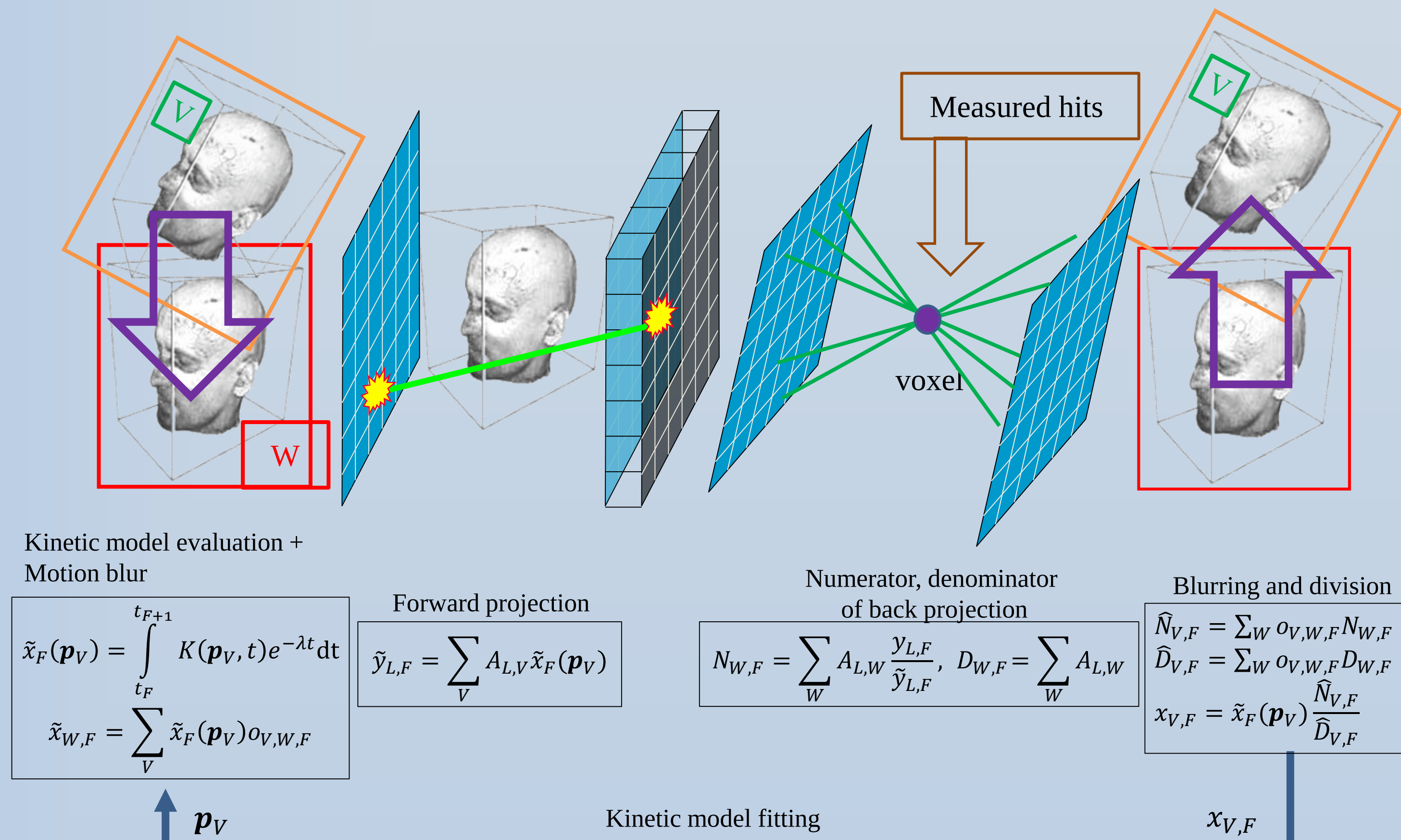
- Motion blur of the tomograph-space numerator and denominator voxel arrays into object-space voxel arrays.

$$\hat{N}_{V,F} = \sum_W o_{V,W,F} N_{W,F} \quad \hat{D}_{V,F} = \sum_W o_{V,W,F} D_{W,F}$$

- Update of voxel activities:

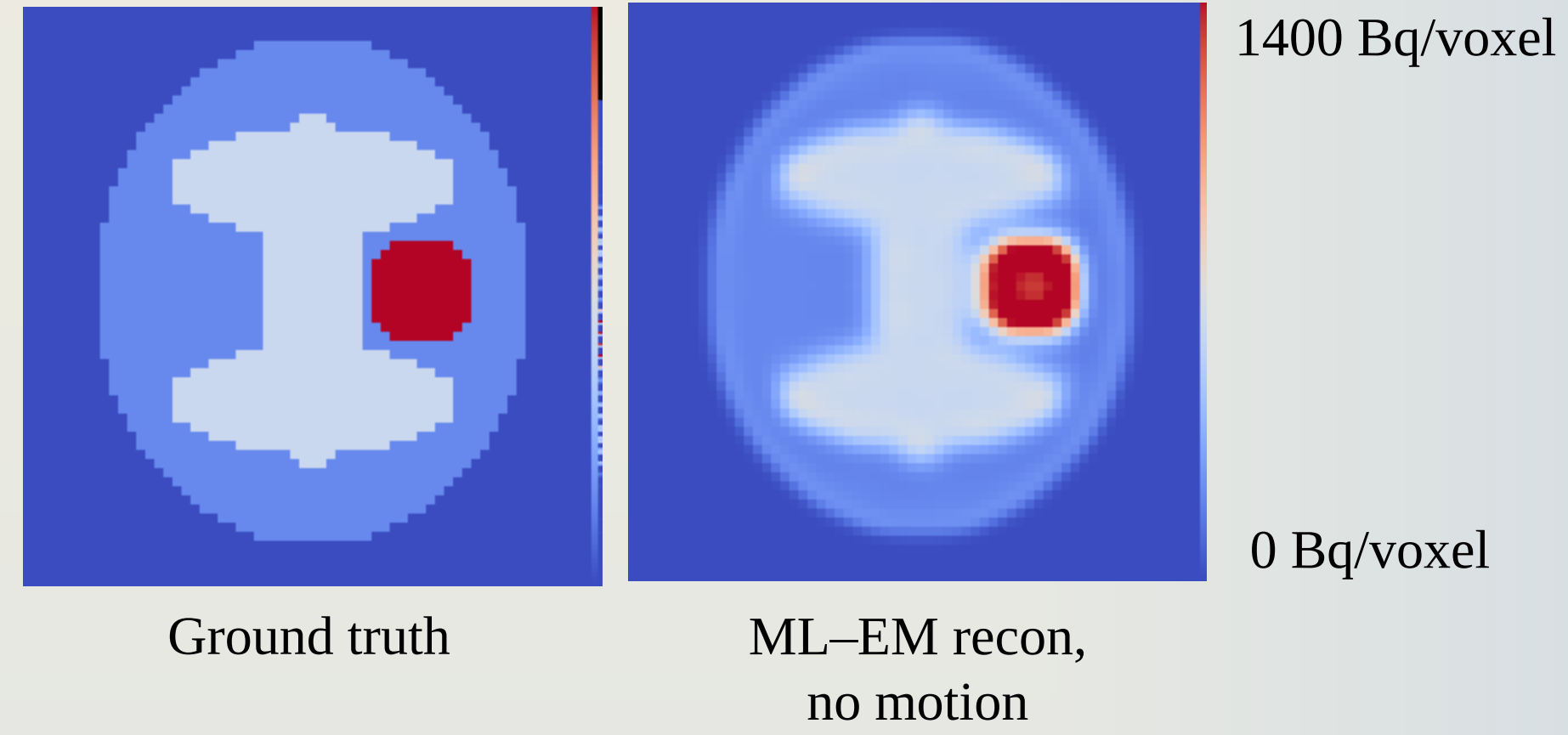
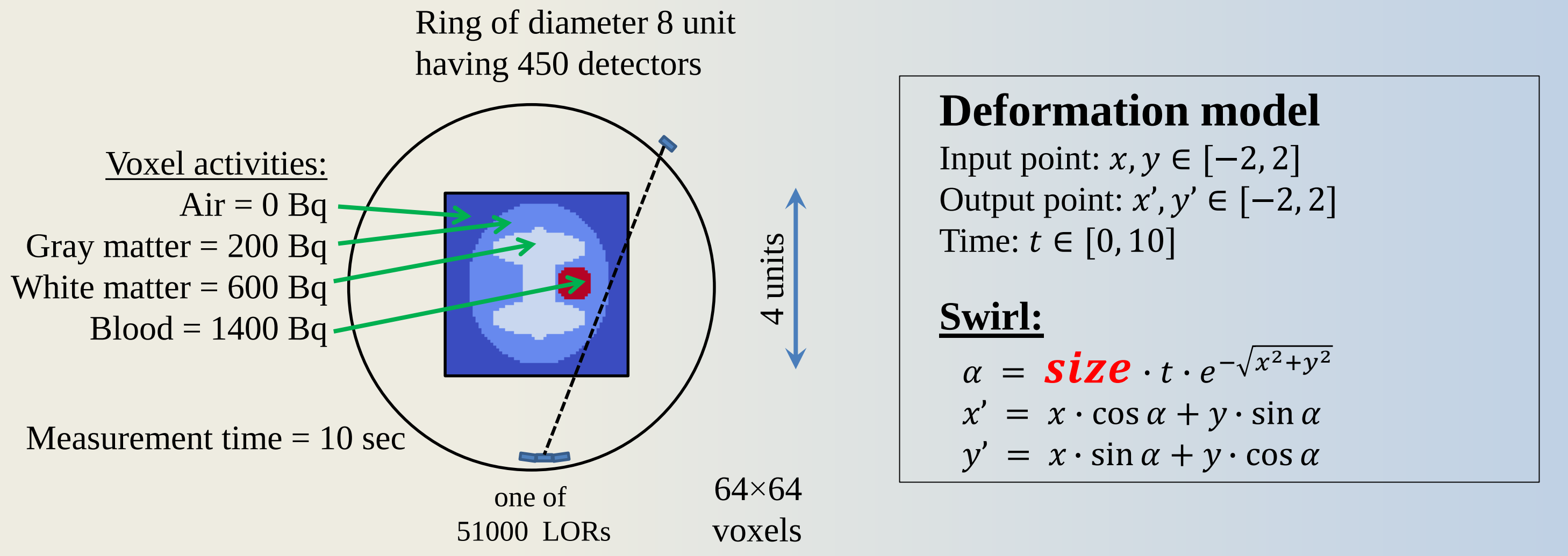
$$x_{V,F} = \tilde{x}_F(\mathbf{p}_V) \frac{\hat{N}_{V,F}}{\hat{D}_{V,F}}$$

- Fitting the Kinetic model parameters  $\mathbf{p}_V$  to  $x_{V,F}$  for each object-space voxel  $V$ .

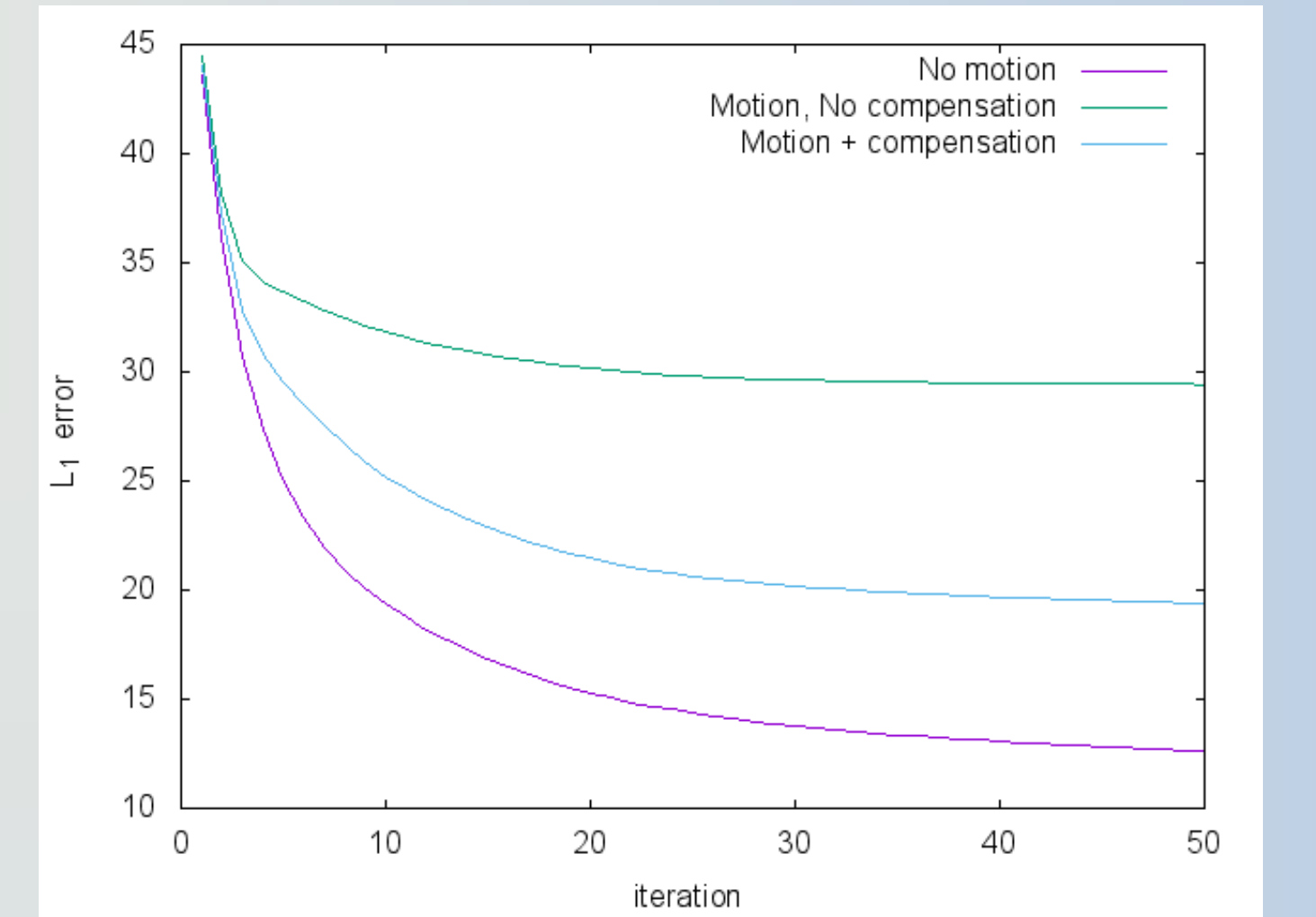
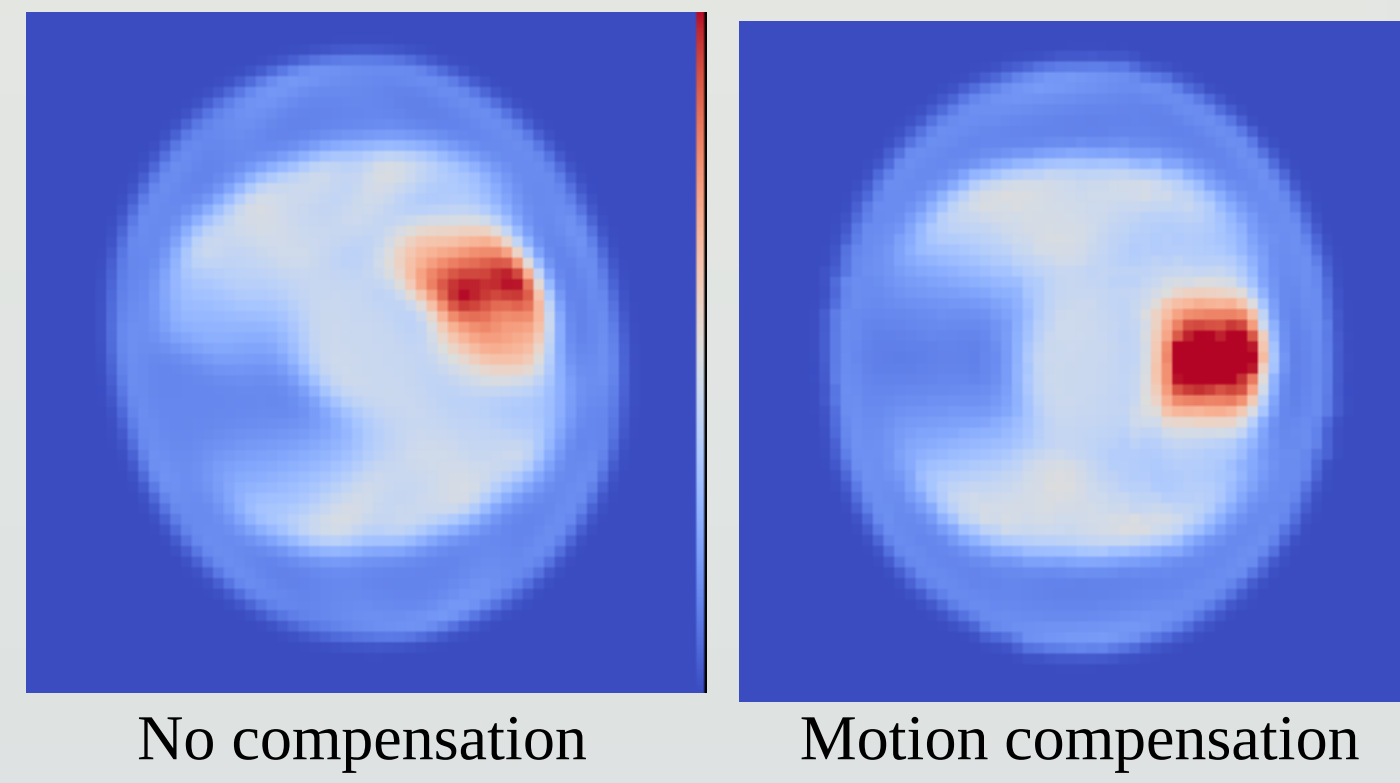


**Observation:** The reconstruction algorithm stays the same, just 3 voxel-space blurring should be added in every iteration.

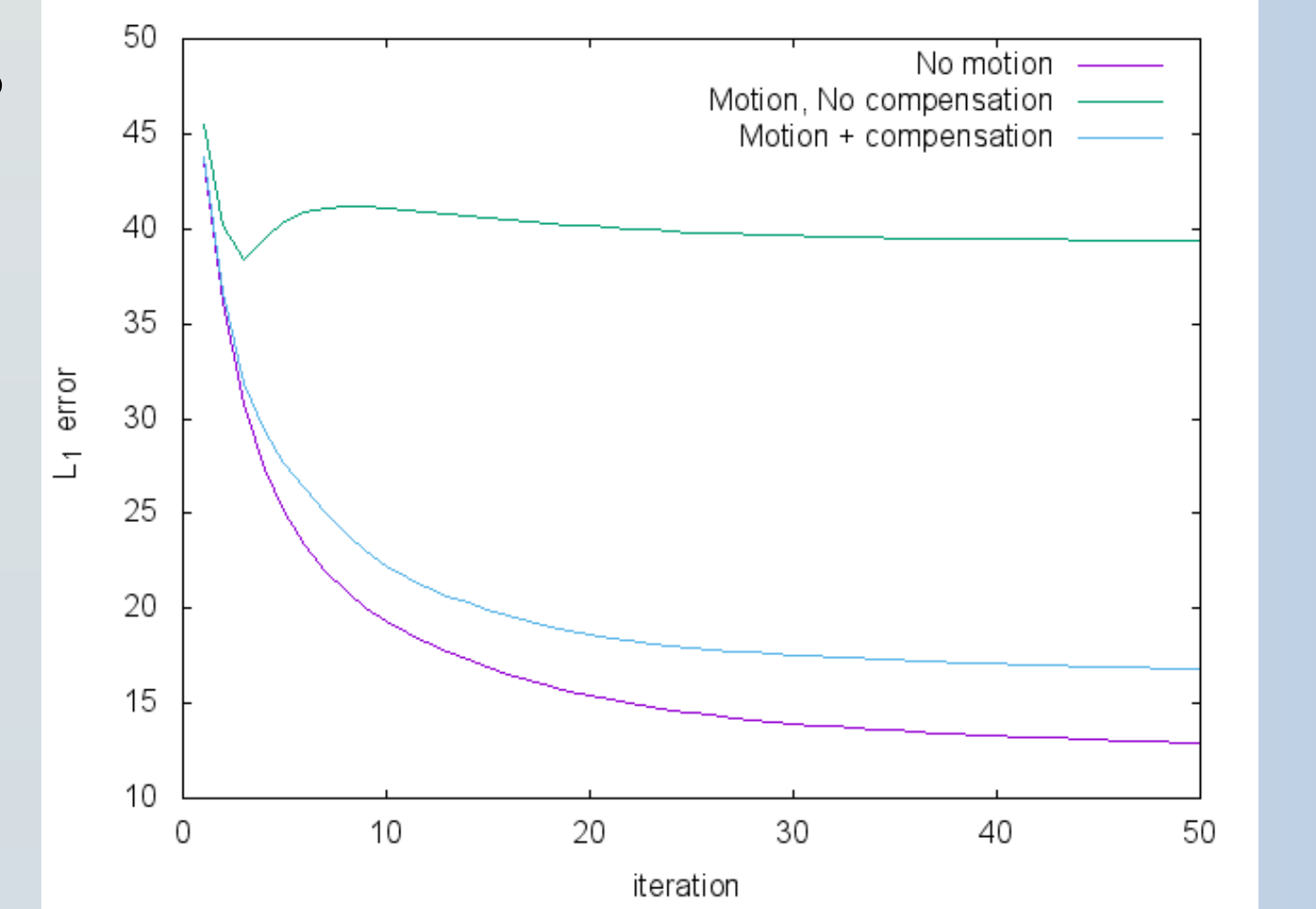
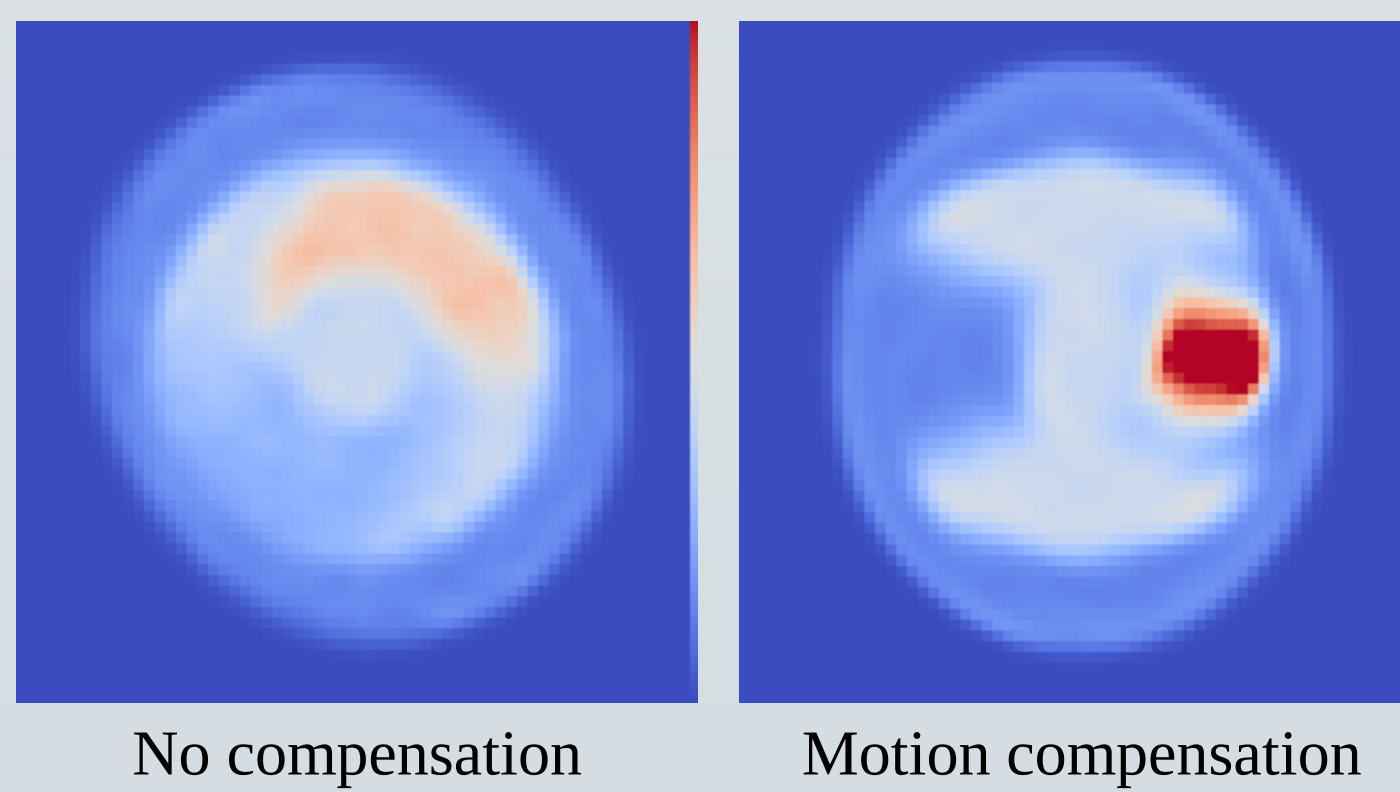
## Results: 2D brain phantom



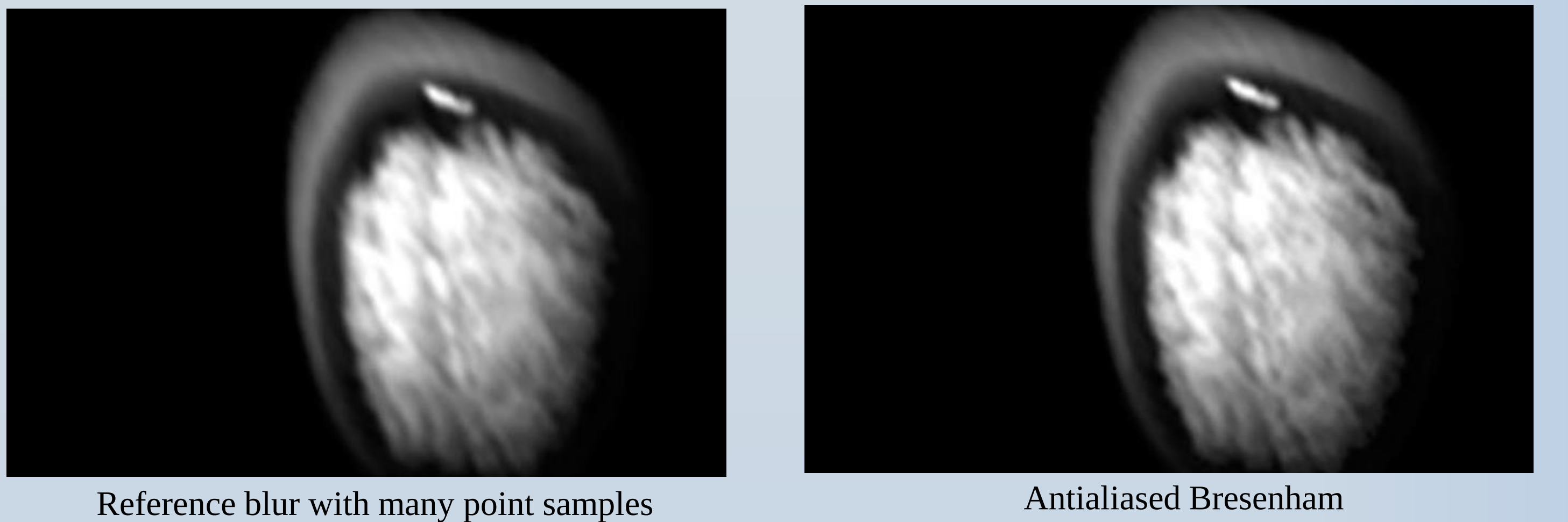
### Small (size=0.2) deformation, 1 frame



### Large (size=0.5) deformation, 5 frames



### Motion blur: Antialiased Bresenham line drawing on GPU



## Conclusions

By adding three motion blurs to the ML-EM reconstruction pipeline, it is possible to compensate larger, even not rigid body motions without the dramatic increase of the number of frames. The blurring operation can be well supported by the GPU and its extra time is negligible compared to the other steps.

## References

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## Acknowledgements

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