

Stability of snaking trailers

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Abstract: The mechanical model of a two-wheeled trailer is constructed to describe the spatial motion and the roll-over of the trailer. The governing equations are derived with special attention to the non-smoothness and non-linearities that strongly affect the dynamics of the trailer. The presence of unstable limit cycles related to these properties makes the vibrations of the trailers intricate from practical point of view. Bistable parameter domains may be observed where large amplitude oscillations and the stable rectilinear motion coexist. The aim of the study is to investigate the effects of the parameters (such as payload position, the stiffness and damping of the suspension and speed) on the linear stability and on the non-linear vibrations. The analysis is based both on analytical and numerical techniques.

1. Introduction

The instability of towed vehicles plays a huge role in road transport. If the trailer or semi-trailer is towed within a certain velocity range and it is badly loaded, the vehicle may start a snaking motion [1, 2]. This motion can even lead to the roll-over of the vehicle causing a serious accident. Therefore, the analysis of the motion of towed vehicles is of utmost importance.

The contents of the paper is the following. The mechanical model of two-wheeled trailers together with the governing equations are introduced in Sec. 2. The linear stability analysis is presented in Sec. 3. The effect of two parameters on the linear stability of the rectilinear motion is presented. In Sec. 4, the periodic solutions of the system are followed using *DDE Biftool*, a *Matlab* package for bifurcation analysis. Finally, comparison between the results of numerical simulation and the bifurcation analysis is presented in Sec. 5.

It is shown that the system can exhibit large amplitude rocking motion. Based on the bifurcation diagrams, one can conclude that large amplitude stable and unstable periodic motions coexist with the stable rectilinear motion. Subcritical bifurcations indicate dangerous system configuration from an engineering point of view.

2. Mechanical model and governing equations

The mechanical model of snaking trailers can be seen on Fig. 1. The trailer is towed with a constant longitudinal speed v . The centre points of the right and the left wheel are denoted

by R and L, while the contact points between the tyres and the ground are denoted by T_R and T_L . The towing length is l , the track width is $2b$. The centre of mass is positioned in point C whose position can be described with parameters e and f . The overall stiffness and the damping of the wheel suspension and tyres are k and c . A previous model showed that the linear stability does not depend on the towing velocity if the lateral stiffness of the king pin at A is very large [3]. Therefore, a spring and a damping are placed between the towing point and point A with a spring stiffness of k_1 and damping c_1 . The mass of the trailer is denoted by m and the mass moment of inertia is Θ_C .

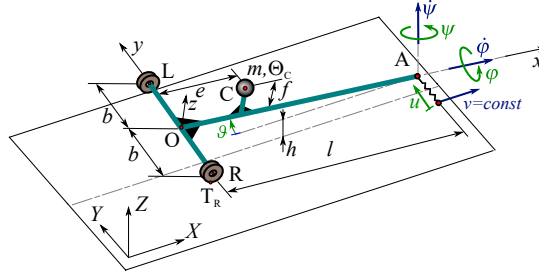


Figure 1. The mechanical model of snaking trailers.

The motion of the trailer can be described with the yaw angle ψ , the pitch angle ϑ , the roll angle φ and the lateral displacement of the king pin u . Therefore, the system has $n = 4$ degrees of freedom. The vector of the generalized coordinates is

$$\mathbf{q} = \begin{bmatrix} \psi & \vartheta & \varphi & u \end{bmatrix}^T. \quad (1)$$

The equations of motion can be derived with the Lagrange equation of the second kind:

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_k} - \frac{\partial T}{\partial q_k} = Q_k, \quad k = 1 \dots n, \quad (2)$$

where T is the kinetic energy of the system, Q_k is the k^{th} component of the generalized force and q_k is the k^{th} generalized coordinate. One can differentiate two coordinate systems: the coordinate system fixed to the ground is denoted by (X, Y, Z) and the coordinate system fixed to the trailer is denoted by (x, y, z) , see Fig. 1.

The kinetic energy of the system can be calculated as

$$T = \frac{1}{2} m v_C^2 + \frac{1}{2} \boldsymbol{\omega}^T \cdot \boldsymbol{\Theta}_C \cdot \boldsymbol{\omega}. \quad (3)$$

The velocity of the towing point in the ground fixed coordinate system is

$$\mathbf{v}_A = \begin{bmatrix} v & \dot{u} & 0 \end{bmatrix}_{(X,Y,Z)}^T. \quad (4)$$

The velocity of the centre of mass and its absolute value can be calculated as

$$\mathbf{v}_C = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{AC}, \quad v_C = |\mathbf{v}_C|, \quad (5)$$

where the angular velocity of the system is

$$\boldsymbol{\omega} = \begin{bmatrix} \dot{\varphi} - \dot{\psi} \sin \vartheta \\ \dot{\vartheta} \cos \varphi + \dot{\psi} \cos \vartheta \sin \varphi \\ \dot{\psi} \cos \vartheta \cos \varphi - \dot{\vartheta} \sin \varphi \end{bmatrix}_{(x,y,z)} \quad (6)$$

and the position vector from point A to point C is

$$\mathbf{r}_{AC} = \begin{bmatrix} -l + e & 0 & f \end{bmatrix}_{(x,y,z)}^T. \quad (7)$$

The mass moment of inertia matrix is considered in the following form:

$$\boldsymbol{\Theta}_C = \begin{bmatrix} \Theta_{C,x} & 0 & 0 \\ 0 & \Theta_{C,y} & 0 \\ 0 & 0 & \Theta_{C,z} \end{bmatrix}_{(x,y,z)} = \begin{bmatrix} \frac{(4b^2+4f^2)m}{12} & 0 & 0 \\ 0 & \frac{(4f^2+l^2)m}{12} & 0 \\ 0 & 0 & \frac{(4b^2+l^2)m}{12} \end{bmatrix}. \quad (8)$$

The generalized force can be derived from the virtual power δP :

$$\delta P = \mathbf{G} \cdot \delta \mathbf{v}_C + \mathbf{F}_{R_{\text{tyre}}} \cdot \delta \mathbf{v}_{T_R} + \mathbf{F}_{R_{\text{susp}}} \cdot \delta \mathbf{v}_R + \mathbf{F}_{L_{\text{tyre}}} \cdot \delta \mathbf{v}_{T_L} + \mathbf{F}_{L_{\text{susp}}} \cdot \delta \mathbf{v}_L + \mathbf{F}_{A_{\text{lat}}} \cdot \delta \mathbf{v}_A, \quad (9)$$

where $\mathbf{G}$ is the gravitational force, $\mathbf{F}_{L_{\text{tyre}}}$ and $\mathbf{F}_{R_{\text{tyre}}}$ are the tyre forces acting at points T_L and T_R , respectively. $\mathbf{F}_{L_{\text{susp}}}$ and $\mathbf{F}_{R_{\text{susp}}}$ are the forces acting on the chassis of the trailer at points L and R due to the radial deformation of the tyres and the deformation of the suspension system. $\mathbf{F}_{A_{\text{lat}}}$ is the lateral force acting at point A. The active forces acting on the trailer are shown on Fig. 2.

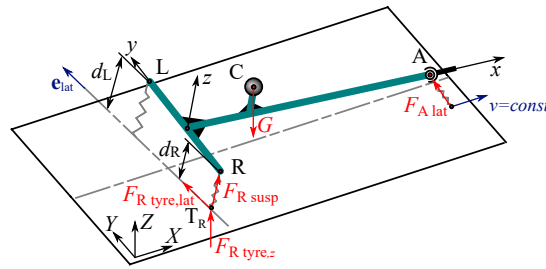


Figure 2. The active forces acting on the trailer.

The gravitational force can be calculated as

$$\mathbf{G} = \begin{bmatrix} 0 & 0 & -mg \end{bmatrix}_{(X,Y,Z)}^T. \quad (10)$$

The direction of the lateral tyre forces can be given using the unit vector $\mathbf{e}_{\text{lat}}$:

$$\mathbf{e}_{\text{lat}} = \begin{bmatrix} \frac{\cos \psi \sin \vartheta \sin \varphi - \sin \psi \cos \varphi}{\sqrt{\sin^2 \vartheta \sin^2 \varphi + \cos^2 \varphi}} \\ \frac{\sin \psi \sin \vartheta \sin \varphi + \cos \psi \cos \varphi}{\sqrt{\sin^2 \vartheta \sin^2 \varphi + \cos^2 \varphi}} \\ 0 \end{bmatrix}_{(X,Y,Z)}, \quad (11)$$

which is the projection of the rotational axis of the wheels y to the plane of the ground (X, Y) , see Fig. 2. Therefore, the tyre forces can be written as

$$\mathbf{F}_{\text{Ltyre}} = F_{\text{Ltyre}} \cdot \mathbf{e}_{\text{lat}}, \quad \mathbf{F}_{\text{Rtyre}} = F_{\text{Rtyre}} \cdot \mathbf{e}_{\text{lat}}. \quad (12)$$

The magnitude of the tyre forces can be calculated with the help of the Magic Formula of Pacejka [4]:

$$F_{\text{tyre,lat}}(\alpha, F_z) = F_z D \sin(C \arctan(B\alpha - E(B\alpha - \arctan(B\alpha)))) , \quad (13)$$

where F_z is the vertical load on the tyre, B is the stiffness factor, C is the shape factor, D is the peak factor and E is the curvature factor. In this current study, these factors were chosen for dry asphalt: $B = 10, C = 1.9, D = 1, E = 0.97$, see [5]. The Magic Formula gives a semi-empirical formula between the lateral and the vertical components of the tyre force and the slip angle α . The side slip angle for the right wheel can be calculated as

$$\alpha_R = -\arctan\left(\frac{\mathbf{v}_{\text{T}_R} \cdot \mathbf{e}_{\text{lat}}}{\mathbf{v}_{\text{T}_R} - (\mathbf{v}_{\text{T}_R} \cdot \mathbf{e}_{\text{lat}})\mathbf{e}_{\text{lat}}}\right). \quad (14)$$

The side slip angle for the left wheel can be calculated similarly.

The force originated in the wheel suspension acts in the z direction, which is not parallel to the Z direction if the trailer is dislocated from its rectilinear motion. Let the distance between the centre point of the right wheel R and the contact point of the wheel T_R be denoted by d_R , see Fig. 2. The equation of the line in direction of z between points R and T_R in the ground fixed coordinate system can be formulated as

$$\mathbf{r}(d_R) = \mathbf{r}_R - \begin{bmatrix} 0 & 0 & d_R \end{bmatrix}^T. \quad (15)$$

Since the vertical component of the position vector of point T_R is zero in the ground fixed coordinate system, the value of d_R can be calculated. Hence, one can calculate the velocity of

point T_R (v_{T_R}), too. The value of d_L and v_{T_L} for the left wheel can be calculated similarly. The suspension forces can be written in the following form:

$$\mathbf{F}_{R_{\text{susp}}} = \begin{bmatrix} 0 & 0 & F_{R_{\text{susp}}} \end{bmatrix}_{(x,y,z)}^T, \quad \mathbf{F}_{L_{\text{susp}}} = \begin{bmatrix} 0 & 0 & F_{L_{\text{susp}}} \end{bmatrix}_{(x,y,z)}^T. \quad (16)$$

The magnitudes of these forces of the suspensions are the sum of the forces acting in the spring of stiffness k and the damping with a damping factor c . In addition, two criteria can be formulated in order to consider the non-smooth effects. Firstly, the force in the suspension can never be positive if the spring is not deformed. Secondly, it has to be taken into account that the forces generated by the dampers can be negative. These criteria can be mathematically described with a so-called Heaviside function, which is formulated in our case as

$$H(x) = \frac{1}{2} \left(1 + \tanh \left(\frac{x}{\varepsilon} \right) \right) \quad (17)$$

with the smoothing parameter ε . The smaller ε is, the more precisely the non-smooth nature of the forces are taken into account. With this Heaviside function, all the motion states can be described with the same set of equations. Thus, the force acting in the suspension at the right wheel is

$$F_{R_{\text{susp}}} = ((L_{R,0} - d_R)k + (v_{T_R,z} - v_{R,z})c) \cdot H(L_{R,0} - d_R) \cdot H((L_{R,0} - d_R)k + (v_{T_R,z} - v_{R,z})c), \quad (18)$$

where the parameter $L_{R,0}$ is the free length of the spring and $v_{T_R,z}$ and $v_{R,z}$ are the components of the velocities in the z direction. The force originated in the left wheel suspension can be calculated by an analogous way.

The velocities of the centre point of the right and the left wheels are

$$\mathbf{v}_R = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{AR}, \quad \mathbf{v}_L = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{AL}, \quad (19)$$

where the position vectors are

$$\mathbf{r}_{AR} = \begin{bmatrix} -l & -b & 0 \end{bmatrix}_{(x,y,z)}^T, \quad \mathbf{r}_{AL} = \begin{bmatrix} -l & b & 0 \end{bmatrix}_{(x,y,z)}^T. \quad (20)$$

The lateral force acting at point A can be calculated as follows:

$$\mathbf{F}_{A_{\text{lat}}} = \begin{bmatrix} 0 & -s_1 u - k_1 \dot{u} & 0 \end{bmatrix}_{(X,Y,Z)}^T. \quad (21)$$

By substituting the forces and the velocities into Eq. (3) and (9) and then into (2), the governing equations can be derived. The equations of motion can be written in the following form: $\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C} = \mathbf{0}$, where $\mathbf{M}$ is the mass matrix and $\mathbf{C}$ is a vector containing the remaining parts of the equation including several non-linearities.

3. Linear stability analysis

By means of the linear stability analysis, the small vibrations around the rectilinear motion are investigated. For this, the governing equations are linearised around the rectilinear motion $\mathbf{q}_0 = \mathbf{0}$, which corresponds to

$$\psi(t) \equiv \psi_0 = 0, \quad \vartheta(t) \equiv \vartheta_0 = 0, \quad \varphi(t) \equiv \varphi_0 = 0, \quad u(t) \equiv u_0 = 0. \quad (22)$$

The free length of the right and the left springs in the suspensions are assumed to be equivalent, thus $\varphi(t) \equiv \varphi_0 = 0$ is true if we choose

$$L_{R,0} \equiv L_{L,0} = h + \frac{mg(l-e)}{2kl}. \quad (23)$$

The linearised equations of motion can be written as

$$\mathbf{M}_{\text{lin}} \ddot{\mathbf{q}} + \mathbf{C}_{\text{lin}} \dot{\mathbf{q}} + \mathbf{K}_{\text{lin}} \mathbf{q} = \mathbf{0}, \quad (24)$$

where $\mathbf{M}_{\text{lin}}$ is the mass matrix, $\mathbf{C}_{\text{lin}}$ is the damping matrix and $\mathbf{K}_{\text{lin}}$ is the stiffness matrix of the linearised system :

$$\mathbf{M}_{\text{lin}} = \mathbf{M}|_{\mathbf{q}_0=\mathbf{0}}, \quad \mathbf{C}_{\text{lin}} = \left. \frac{\partial \mathbf{C}}{\partial \dot{\mathbf{q}}} \right|_{\mathbf{q}_0=\mathbf{0}}, \quad \mathbf{K}_{\text{lin}} = \left. \frac{\partial \mathbf{K}}{\partial \mathbf{q}} \right|_{\mathbf{q}_0=\mathbf{0}}. \quad (25)$$

These matrices are given in [3]. The linearised system can be separated into two subsystems: one differential equation can be disjointed, namely the pitch motion can be analysed alone ($n = 1$ DoF subsystem), while the remaining equations are coupled ($n = 3$ DoF subsystem). The stability of the rectilinear motion can be investigated with the help of these equations using the Routh-Hurwitz criteria. The parameter values which are unchanged in the study can be seen in Tab. 1.

Table 1. Parameter values which are unchanged in the study.

l [m]	b [m]	h [m]	e [m]	m [kg]	c_1 [Ns/m]
3	0.8	0.5	0.9	3000	100

On the one hand, no closed form formula can be derived for the critical velocity value for the general case. However, for a special case, when the lateral stiffness is assumed to be very large ($k_1 \rightarrow \infty$) and the damping factor is zero ($c = 0$), the critical values for the stiffness k can be calculated [3]. On the other hand, the linear stability can be investigated numerically. Fig. 3 shows the real part of the rightmost characteristic root versus the towing speed v . Since for $v < 3.5$ m/s, the real part of the rightmost characteristic root is negative, the rectilinear motion is linearly stable (blue continuous line). For $v > 3.5$ m/s, there exist a

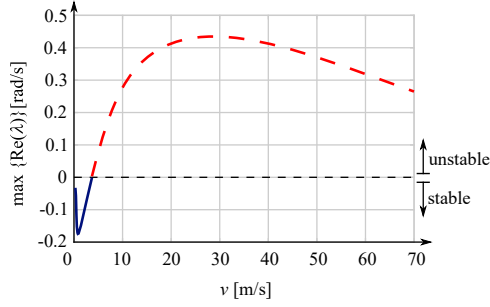


Figure 3. The real part of the rightmost characteristic root as a function of v . The continuous line corresponds to stable, the dashed line corresponds to unstable rectilinear motion. Parameter values: $f = 1.2$ m, $k = 60$ kN/m, $c = 4$ kNs/m, $k_1 = 10$ kN/m.

complex conjugate root pair whose real part is positive; therefore, the rectilinear motion is linearly unstable, dynamic stability loss occurs (red dashed line).

In addition, one can also investigate the effect of different parameters on the linear stability of the rectilinear motion. Panel a) of Fig. 4 shows the effect of the damping c , while panel b) shows the effect of the stiffness k versus the towing speed v . The light grey areas correspond to linearly unstable rectilinear motion, where snaking, rocking, roll-over of the trailer may appear; the white areas correspond to linearly stable rectilinear motion.

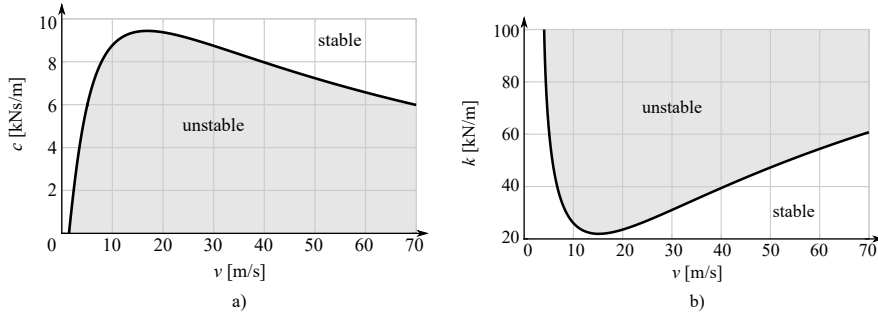


Figure 4. The effect of a) the damping and b) the stiffness on the linear stability of the rectilinear motion. Parameter values: $f = 1.2$ m, $k_1 = 10$ kN/m, a) $k = 60$ kN/m, b) $c = 4$ kNs/m.

4. Non-linear analysis

The non-linear behaviour of the trailer is investigated with the help of a *Matlab* package for bifurcation analysis, *DDE Biftool* [6]. During the analysis, the Hopf bifurcations corre-

sponding to the linear stability boundaries were located and the branches of the emerging periodic solutions were followed.

In the case when the center of gravity of the trailer is close to the wheels, i.e. $e < 1$ m, the trailer can start rocking from one of its wheels to the other above a certain towing speed. Thus, the non-smooth nature of the vertical forces acting on the wheels has a strong effect, and numerical problems can occur during the continuation. As a result, sudden changes can be observed in the bifurcation branches, see, for example, the maximal value $\varphi_{\max}$ of the roll angle on Fig. 5. Between 8 m/s and 54 m/s, the minimum of the forces acting in the suspensions is zero, i.e. there are some time instants where one of the wheels leaves the ground. Below 8 m/s and above 54 m/s, both wheels are on the ground, there are no numerical problems and the branch is smooth. S_1 and S_2 are points for which numerical simulations are run, see Sec. 5.

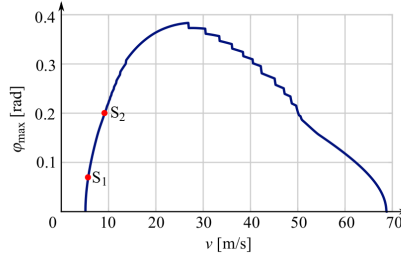


Figure 5. The amplitude of the rolling angle φ by means of the towing velocity v . Parameter values: $f = 1.2$ m, $k = 60$ kN/m, $c = 6$ kNs/m, $k_1 = 10$ kN/m.

The smoothness of the bifurcation branches depends on several parameters. First of all, the periodic solution is calculated in specific mesh points with number N . The greater N is, the more smooth the branch becomes as can be seen in panel a) of Fig. 6. Since having a large N value makes the continuation slow, N is chosen to be 144 in this study. As a result, the continuation runs in an acceptable speed and the branch is smooth enough. Secondly, the degree p of the interpolation polynomial is another key factor. Panel b) of Fig. 6 shows the effect of the value of parameter p . It can be seen that the larger p is, the smoother the branch is. However, increasing the value of p leads to significantly growing computational time. Since in our case, the impact between the ground and one of the wheels happens very suddenly, p is chosen to be 1, i.e. linear interpolation is used. Thirdly, the smoothing parameter ε of the so-called Heaviside function of Eq. (17) has to be as small as possible in order to switch between motion states in one time step. We use $\varepsilon = 10^{-7}$ in our analysis.

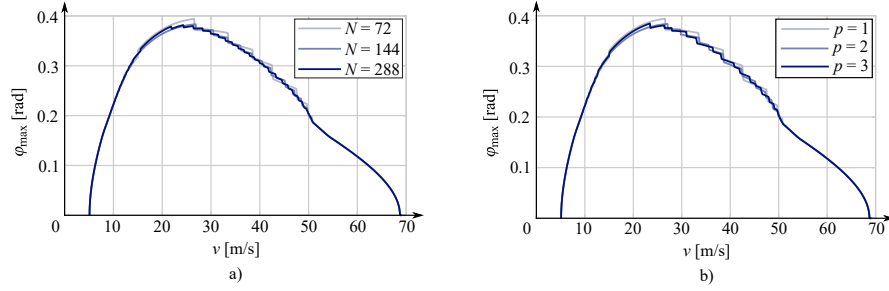


Figure 6. The effect of the value of a) parameter N , b) parameter p on the amplitude of the rolling angle φ . Parameter values: $f = 1.2$ m, $k = 60$ kN/m, $c = 6$ kNs/m, $k_1 = 10$ kN/m, a) $p = 1$, b) $N = 72$.

The periodic solutions and the effect of parameter f on the linear stability are shown together in Fig. 7. The light grey areas correspond to linearly unstable rectilinear motion. One can observe how the amplitude of the roll angle changes with increasing value of parameter f .

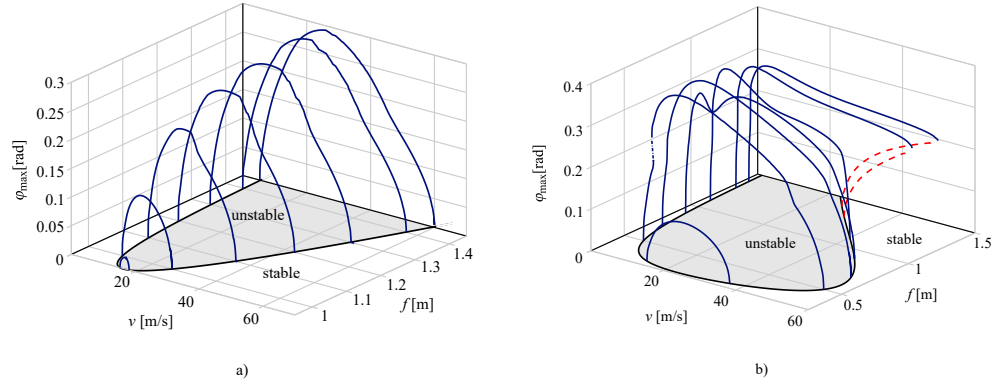


Figure 7. Periodic solution branches and the effect of parameter f for two different parameter sets. Continuous lines correspond to stable, dashed lines correspond to unstable branches. Parameter values: $k = 60$ kN/m, a) $c = 6$ kNs/m, $k_1 = 10$ kN/m; b) $c = 1.9$ kNs/m, $k_1 = 100$ N/m.

In case of $c = 6$ kNs/m and $k_1 = 10$ kN/m, the unstable region grows when the value of f increases and all the periodic solution branches are stable, see panel a) of Fig. 7. However, in case of smaller damping value c and the lateral stiffness k_1 , unstable branches appear above $f = 1.2$ m, see panel b) of Fig. 7. That is, subcritical bifurcation can be found by following the branch of periodic solutions from the second Hopf bifurcation point. One can also observe a special characteristics of $\varphi_{\max}$ if $f = 0.9$ m. The stippled line in case of

$f = 0.5$ m indicates numerical problems in the continuation.

The bifurcation diagram of the system for $f = 1.2$ m can be seen on Fig. 8. From about $v = 35.7$ m/s, the rectilinear motion becomes stable, and an unstable periodic orbit exists together with the large amplitude stable periodic motion. Therefore, if a large enough perturbation is applied for the system, dangerous rocking motion may occur.

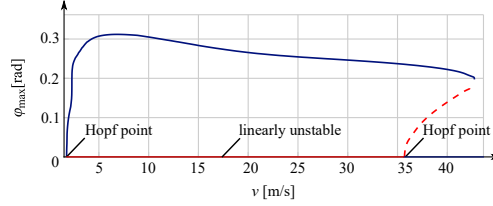


Figure 8. Bifurcation diagram in the plane of the amplitude of the roll angle φ and the towing speed v . Stable and unstable orbits are denoted by continuous and dashed lines, respectively. Parameter values: $f = 1.2$ m, $k = 60$ kN/m, $c = 1.9$ kNs/m, $k_1 = 100$ N/m.

5. Numerical simulation

The critical velocity values given by the linear stability analysis and the periodic solution given by the bifurcation analysis can be verified by means of numerical simulations. For these numerical simulations, fourth order Runge-Kutta method was used with fix time step. Different parameter values and initial conditions were used. Here, two simulation results are presented with their comparison to the results of the bifurcation analysis. Panel a) of Fig. 9 shows the time histories of the generalized coordinates and the vertical forces acting on the right and the left wheels at $v = 5.83$ m/s (point S_1 on Fig. 5). As can be seen, the vertical forces remain positive for the whole period, i.e. no rocking motion is present. For $v = 8.95$ m/s (point S_2 on Fig. 5), the motion tends to a large amplitude rocking motion, see panel b) of Fig. 9. For both cases, the simulation and bifurcation analysis results are the same.

The rocking and snaking motion of the trailer is illustrated in Fig. 10 via snapshots made from an animation. As can be seen, the trailer rocks from one of its wheels to the other while making a snaking motion.

6. Conclusions

A mechanical model was introduced for the investigation of the stability of two-wheeled trailers. The dependence of the linear stability of the rectilinear motion on the towing speed was shown. In addition, the effects of two parameters (the damping and the stiffness of

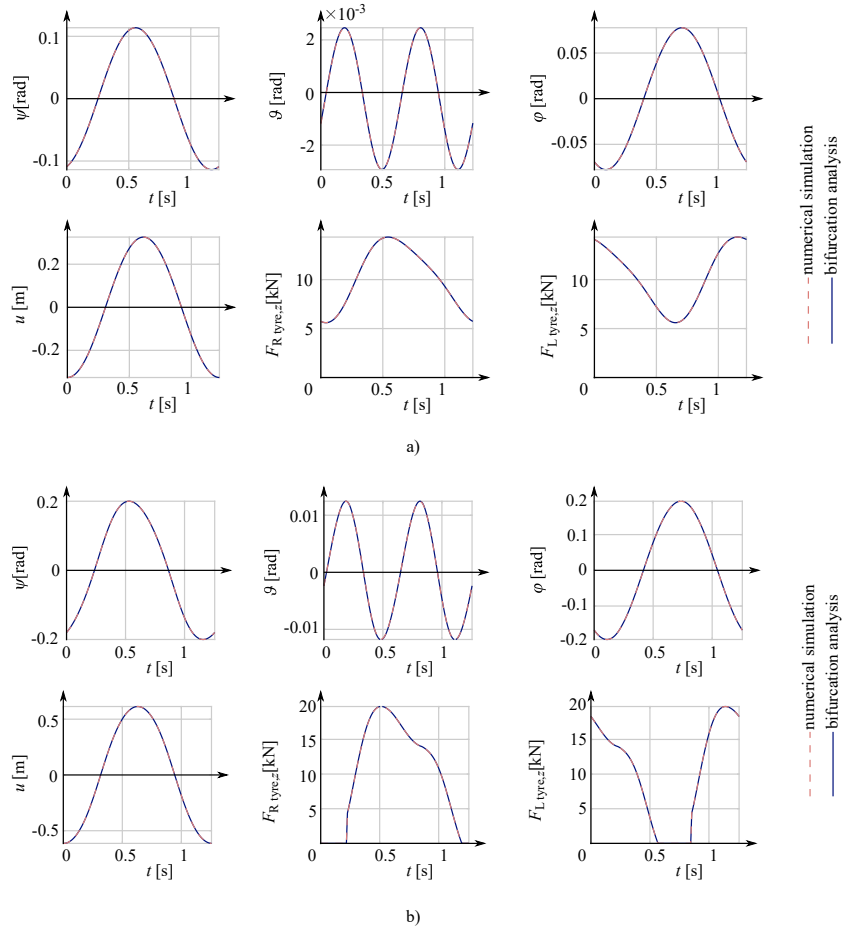


Figure 9. The time histories of the generalized coordinates and the vertical forces acting on the right and the left wheels: a) no rocking motion, b) rocking motion. Pink dashed lines correspond to numerical simulation results, blue continuous lines correspond to the results of bifurcation analysis. Parameter values: $f = 1.2$ m, $k = 60$ kN/m, $c = 6$ kNs/m, $k_1 = 10$ kN/m, a) $v = 5.83$ m/s, b) $v = 8.95$ m/s.

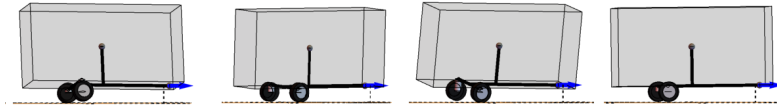


Figure 10. Illustration of the rocking and snaking motion: snapshots from animation in case of $v = 8.95$ m/s.

the suspensions) on the linear stability were presented. Periodic solutions were detected numerically. Based on the bifurcation diagrams, one can conclude that the system can exhibit large amplitude rocking motion. It was also shown that unstable periodic orbits may exist representing a safety hazard.

Acknowledgments

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