

MODELLING AND SIMULATION OF ROCKING SUITCASES

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ABSTRACT. The rocking motion of two-wheeled suitcases is investigated. A rigid body mechanical model of the suitcase is constructed. All of the possible motion states (both wheels on the ground, one wheel on the ground, none of the wheels on the ground) are taken into account. The switching between the motion states is accomplished by a simple impact model. The motion of the suitcase is investigated through numerical simulations, furthermore the domain of the attraction of the stable rectilinear motion is identified. The model is partly validated by experiments.

KEYWORDS: stability, suitcase, non-smooth system, non-linear system.

1. INTRODUCTION

The rocking motion of two-wheeled suitcases is an annoying problem since it makes the use of suitcases uncomfortable in the everyday life. Large enough perturbations of the rolling suitcase can lead to the rocking motion, when the suitcase jumps from one of its wheels to the other, and in crucial cases the suitcase overturns. The topic is researched even nowadays (see [1]), although papers focusing on rocking suitcases were already published in the 90's (see [2–4]).

The interest of researchers is supported by the fact that similar stability problem of trailers exists in vehicle dynamics (see, for example, [5]). From mechanical point of view, the analysis of the rocking suitcase problem is very complex. To begin with, the system is non-smooth since different equations describe the different motion states. Additionally, in case of contacting wheel, the spatial motion of the suitcase is influenced by the kinematic constraints, which makes the derivation of the equation of motion more complicated.

Due to this complexity, former studies on the field do not take into account all of the motion states of the suitcase, namely, the case when none of the wheels has contact with the ground is neglected. In this study, a mechanical model of the rocking suitcase is constructed in which all motion states are considered. Experiments are also shown that confirm the presence of these motion states.

The contents of the paper is the following. The simplified mechanical model is introduced in Sec. 2. The different motion states together with the corresponding number of the generalized coordinates are summarized in Sec. 3. The motion of the system is analysed through numerical simulations in Sec. 4. Based on these simulations, experiments are carried out in Sec. 5. The comparison of the simulation and the measurement results can be found in Sec. 6.

2. MECHANICAL MODEL

The simplified mechanical model is depicted in Fig.1. The suitcase is pulled in the X direction with constant speed v by the ball-joint at A. The body of the suitcase is assumed to be rigid and it is modelled by means of massless rods and by a lumped mass m at the center of mass C of the suitcase. The position of the lumped mass is characterized with parameters e and f . The length and the width of the suitcase are described by the parameters l and $2b$, respectively, while h is the distance between the towing point A and the ground. The radius of the wheel (r) is assumed to be zero, which is a suitable approximation in our simplified model and it is also used in several studies (see, for example, [1]). On the one hand, the radius of the wheel is relatively small to the other dimensions of the suitcase and trajectory of the wheel center points and the paths of the contact points are very close to each other. On the other hand, the gyroscopic effect acting on the wheels is negligible since the mass moment of inertia of the wheels and the rotational speed are very small.

3. MOTION STATES

One can differentiate four different motion states of the suitcase: both wheels have contact with the ground, one wheel (right or left wheel) is on the ground, none of the wheels is on the ground. See Fig. 2, where the motion states are given together with the transitions between them. The different motion states correspond to 1, 3 and 6 dimensional state spaces, respectively.

When both wheels are on the ground, the suitcase makes in-plane motion and the equations of motion can be easily derived from the kinematic constraint of rolling. The position of the suitcase is given by the deflection angle ψ (see in Fig.1), and the motion can be described in a one-dimensional phase space.

Having only one wheel on the ground is the most intricate case, since the motion is spatial, while the

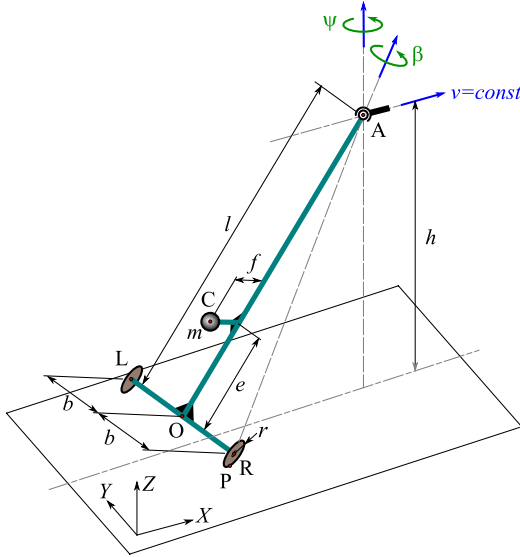


FIGURE 1. The simplified mechanical model of a towed two-wheeled suitcase. The generalized coordinates for the right wheel rolling case.

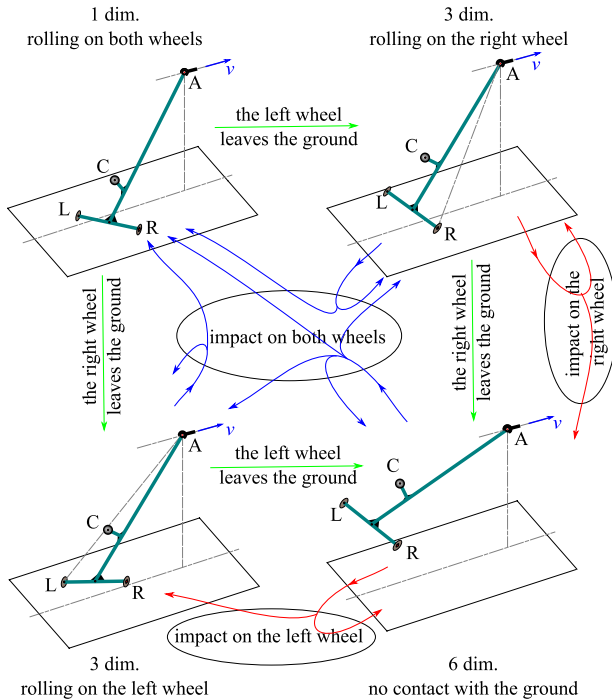


FIGURE 2. The motion states with the corresponding dimensions of the state spaces, and the possible transitions between them.

kinematic constraint of rolling has to be considered. Thus, for example, the equations of motion can be derived with the so-called Routh–Voss-equation. In this case, the governing equations lead to a three-dimensional phase space given by the tilting angle β , the angular velocity $\Omega = \dot{\beta}$ and deflection angle ψ (see Fig. 1).

If none of the wheels have contact with the ground, the motion is spatial but the system simplifies to a spatial mathematical pendulum with an extra rota-

tional degree of freedom about the axis determined by the points A and C. Namely, a six-dimensional state space governs the motion.

To make switches between the motion states, the detachments of the wheels from the ground and the impact between the wheels and the ground have to be handled. Namely, if the normal force at one of the wheels that has contact with the ground becomes zero, the wheel can leave the ground, and another motion state will govern the further motion of the suitcase.

Similarly, if one of the non-contacting wheel touches the ground with non-zero normal velocity, impact happens between the wheel and ground. To describe the impact, a complex impact model has to be applied since classical rigid body impact models could not be used to describe the practically observed transitions. Here, we use a spring and a damper as a simple contact element to model the impact of the wheel. The deformations of the springs at the left and the right wheels can be described by the vertical positions z_L and z_R of the wheel centre points. If one of the wheels is on the ground then the contact spring is compressed and the vertical coordinate of the wheel is a small negative value. This small vertical displacement of the wheel centre point is negligible relative to the large scale motion of the suitcase. Moreover, since the dynamic of the impact is much faster than the position change of the body, the simulations of the impact can be carried out in another time scale. This method is often used for such multi-scale systems.

4. NUMERICAL SIMULATIONS

The governing equations and the impact model were implemented in computer code. The implemented equations of motion were rewritten into a system of first order ordinary differential equations. Fourth order Runge–Kutta method was used in these numerical simulations with fixed time step. Since the system is non-smooth and all motion states had different number of degrees of freedom, and consequently different generalized coordinates and generalized velocities, the coupling between the motion states is a complex task. See Fig. 2, where the transitions can be seen with all the possible scenarios.

Simulations were run for different initial conditions ($\beta(t=0) = \beta_0$, $\Omega(t=0) = \Omega_0$ and $\psi(t=0) = \psi_0$ values) given for the left wheel rolling case and for various parameters (e.g. towing velocity v , geometrical parameters l , h , e and f). In our study, a motion is called stable if the simulation ends in the motion state in which both wheels are on the ground. Otherwise, the motion is called unstable, namely, the suitcase overturns.

The time histories for a stable motion can be seen in Fig. 3. The suitcase rolls on its left wheel (panels in the first row), then the right wheel has an impact meanwhile the left wheel is already on the ground (panels in the second row show the deformations z_L and z_R of the left and right wheels, respectively), at

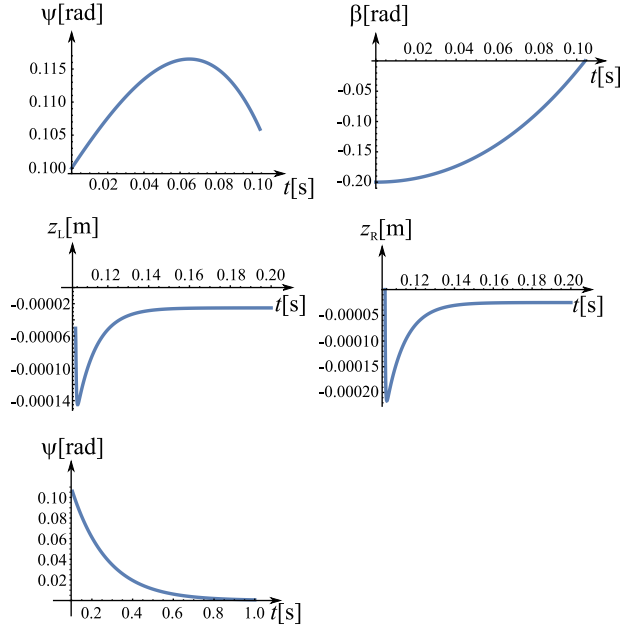


FIGURE 3. Time histories for the initial conditions: $\beta_0 = -0.2$ [rad], $\Omega_0 = 0$ [rad/s], $\psi_0 = 0.1$ [rad], and for parameter values: $l = 0.58$ m, $h = 0.553$ m, $b = 0.16$ m, $e = 0.3$ m, $f = 0.1$ m, $m = 5$ kg, $v = 1$ m/s.

last both wheels have contact with the ground (panel in the third row).

One can also draw the basis of attraction of the rectilinear motion by means of these numerical simulations. In Fig. 4, the results of a simplified mechanical model are shown, where the fourth motion state (no wheels on the ground) was not considered. In the figure, one

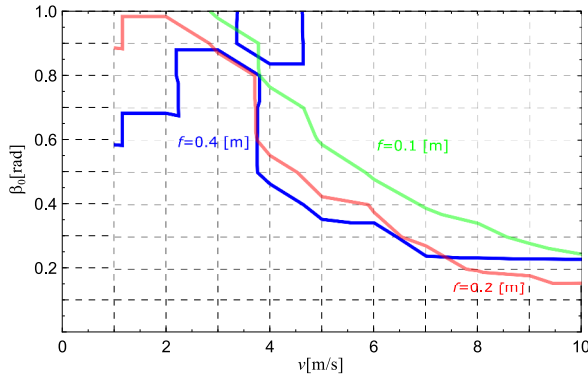


FIGURE 4. The basis of attraction of the rectilinear motion. The parameter values are the following: $l = 1$ m, $h = 0.6$ m, $b = 0.3$ m, $m = 10$ kg. Different colours represent different basis of attraction considering different f values.

can observe the attractive and the repelling regions, i.e. below and above a plotted line, respectively.

Other figures could be depicted considering different geometrical parameter (e , b , h or l) values. If the value of b or l is increased, the attractive region grows as well. In case of increasing e , f or h , the repelling region grows. Due to the non-linearity of the system, there exists such towing velocities in case of a certain

f value, that for specific initial tilting angles β_0 can lead to straight rolling, while a reduced initial angle (smaller perturbation) generates unstable behaviour and the suitcase rolls over, see the blue loop in Fig. 4. Such phenomenon can be caused by the presence of unstable limit cycles, see [6]. Similar behaviour was also observed in our experiments.

5. MEASUREMENTS

For the measurements, a model-based experimental setup was built and was placed on a treadmill while the towing point A was fixed with a ball-joint. The velocity of the treadmill can be varied between 0.5 km/h and 18 km/h. The measurement setup can be seen in Fig. 5.



FIGURE 5. The measurement setup. The parameter values: $l = 0.58$ m, $b = 0.16$ m, $m = 5$ kg. Parameters h , e and f and the velocity v can be varied. Measurements showed that parameters e and f play a huge role in the stability of two-wheeled suitcases.

Two types of measurements were carried out in order to validate the results of the numerical simulations. During the „classical” measurement, the towed structure was perturbed, namely an obstacle was placed onto the treadmill at one of the wheels. The stability of the system could only be determined on statistical basis (out of ten measurements, how many were stable and how many were unstable), since the identical perturbation could not be guaranteed during the measurement. Based on these measurements, we could conclude that parameters e and f play a huge role in the stability of two-wheeled suitcases. The domain of attraction of the rectilinear motion is large even in case of large velocities when the values of parameters e and f are small.

Measurements with OptiTrack motion capture system were also carried out. Four markers were placed on the suitcase (at the towing point A, at the center of mass C and at the left (L) and the right (R) wheels). An obstacle was placed before the left wheel. Four cameras recorded the positions of the markers with respect to time and as a result, the spatial motion of the suitcase was captured. Since this kind of measurement provides us information about the

markers' positions, velocities and accelerations can be computed. As a result, important information can be gained about the level of perturbations with the evaluation of these measurements. Therefore, a more sophisticated identification of the basis of attraction can be carried out.

Snapshots about the measurements is shown in Fig. 6, where the rocking motion can be also observed. The suitcase was perturbed by an obstacle before its left wheel, as a result, its left wheel leaves the ground (1). The suitcase rocks from its left wheel (2) to its right wheel (3). There are time instances when none of the wheels touches the ground (4). After this "flying" motion, the body has an impact with the ground (5), then it overturns (6).

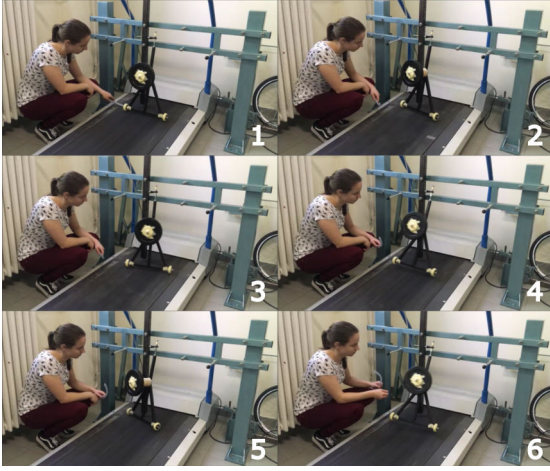


FIGURE 6. Snapshots about the measurement on the treadmill. The suitcase starts to rock and at the end, it overturns.

A stable motion is illustrated in Fig. 7, where the position of the markers are plotted for the measured time interval. It can be seen that the suitcase was perturbed at the left wheel, therefore it jumped from the ground and after falling back it had several impacts with the ground. The maximum of the left wheel's vertical position was about 12 mm, while this value was 8 mm for the right wheel. As it was formally mentioned in Sec. 3, the negative values of the vertical positions of the wheels refer to the elastic deformations of the wheels. After some impacts, the rocking motion of the body dies away, the system stabilizes.

An unstable motion is depicted in Fig. 8. The suitcase rocks from one wheel to the other and the amplitudes grow with respect to time. As a result, the suitcase overturns.

6. COMPARISON

The validation of the model can be done by comparing the simulation results to the measurement ones. Here, we only give a comparison for a specific parameter setup of the suitcase, see Fig. 9. The parameter values of the experiment were: $l = 0.58$ m, $b = 0.16$ m, $e = 0.135$ m, $f = 0.06$ m, $m = 7.2$ kg, $h = 0.553$ m.

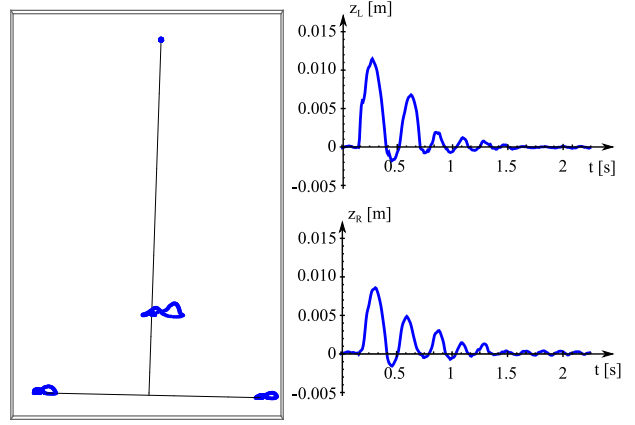


FIGURE 7. Example from an OptiTrack measurement, stable motion. The paths of the markers can be seen on the left figure. z_L and z_R denote the vertical displacement (with respect to time) of the left and the right wheel, respectively.

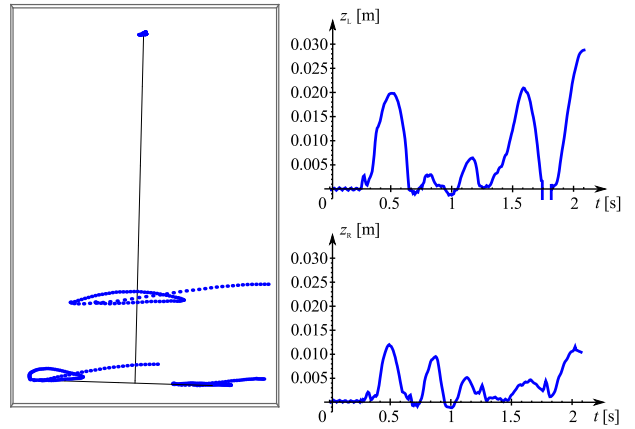


FIGURE 8. Example from an OptiTrack measurement, unstable motion. The paths of the markers can be seen on the left figure. z_L and z_R denote the vertical displacement (with respect to time) of the left and the right wheel, respectively.

In the figure, the result of the simulation is plotted with blue line, the attractive and the repelling regions with respect to the initial angle β_0 can be seen below and above this line. The measurement results are plotted with red dots. The secondary axis at the right hand side of the diagram shows the diameter d_0 of the cylindrical obstacle, which were used to the initiate the rocking motion of the suitcase. In the figure, the diameter is considered to be proportional with the initial tilting angle β_0 .

As it can be seen in the figure, the results of the simulations and the measurements are qualitatively the same. The attractive domain is smaller for larger speeds. Of course, some quantitative difference can be identified in the figure. One of the reason of this difference is that we assumed here that the diameters of the obstacles are linearly proportional with the initial tilting angles. As mentioned before, this assumption is not perfect, and a more sophisticated comparison

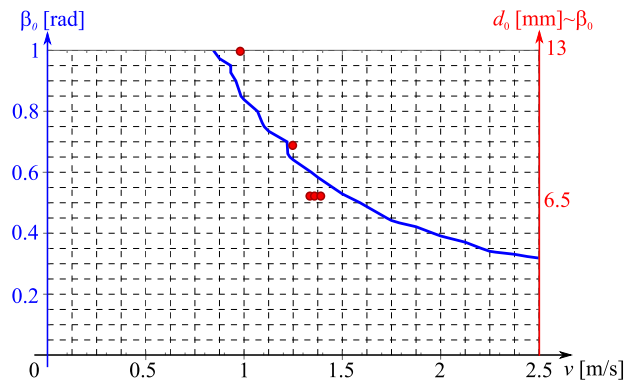


FIGURE 9. Comparison of the results of the numerical simulation and measurements. The simulation result is depicted with the blue line, while the measurement points are the red dots.

of the simulation and measurement results could be done with the evaluation of OptiTrack measurements, which is the task of the future work.

7. CONCLUSIONS

A simplified mechanical model of two-wheeled rocking suitcases was constructed. A summary about the equations of motion of the four different motion states were given, and also a simple impact model was presented. Through numerical simulations important information about the non-linear behaviour of the suitcase was identified. An experimental rig was shown by which the numerical results were partly validated, namely, it was shown that the simulation results are qualitatively correct. Further research on the mechanical model and on the experiment can provide basic rules that can help the design of suitcases having better stability properties.

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Abstract

The stability of suitcases is an important and irritating problem of everyday life as two-wheeled suitcases are often used for travelling. As known, a suitcase can start to rock if large enough perturbation occurs, and in crucial cases the suitcase overturns. The topic is so significant that it is researched in high quality scientific journals even nowadays, see [1, 2]. The interest of researchers is supported by the fact that similar stability problem of trailers exists in vehicle dynamics. In this study, we analyse the system through numerical and experimental methods.

1. Introduction

In mechanical point of view, the analysis of the rocking suitcase problem is very fascinating. On the one hand, the system is non-smooth as different equations describe the different motion states. On the other hand, in case of contacting wheel, the spatial motion of the suitcase is influenced by the kinematic constraints, which makes the derivation of the equation of motion more complicated.

2. Mechanical model

The simplified mechanical model is depicted in Fig.1. The suitcase is pulled with constant speed v by the ball-joint at A. The body of the suitcase is assumed to be rigid and it is modelled by means of massless rods and by a lumped mass m at its center of mass C. One can describe the position of the mass with parameters e and f . The length and the width of the suitcase are described with the parameters l and $2b$, respectively, while h is the distance between the towing point A and the ground. The radius of the wheel (r) is assumed to be zero, which is a suitable approximation in our simplified model.

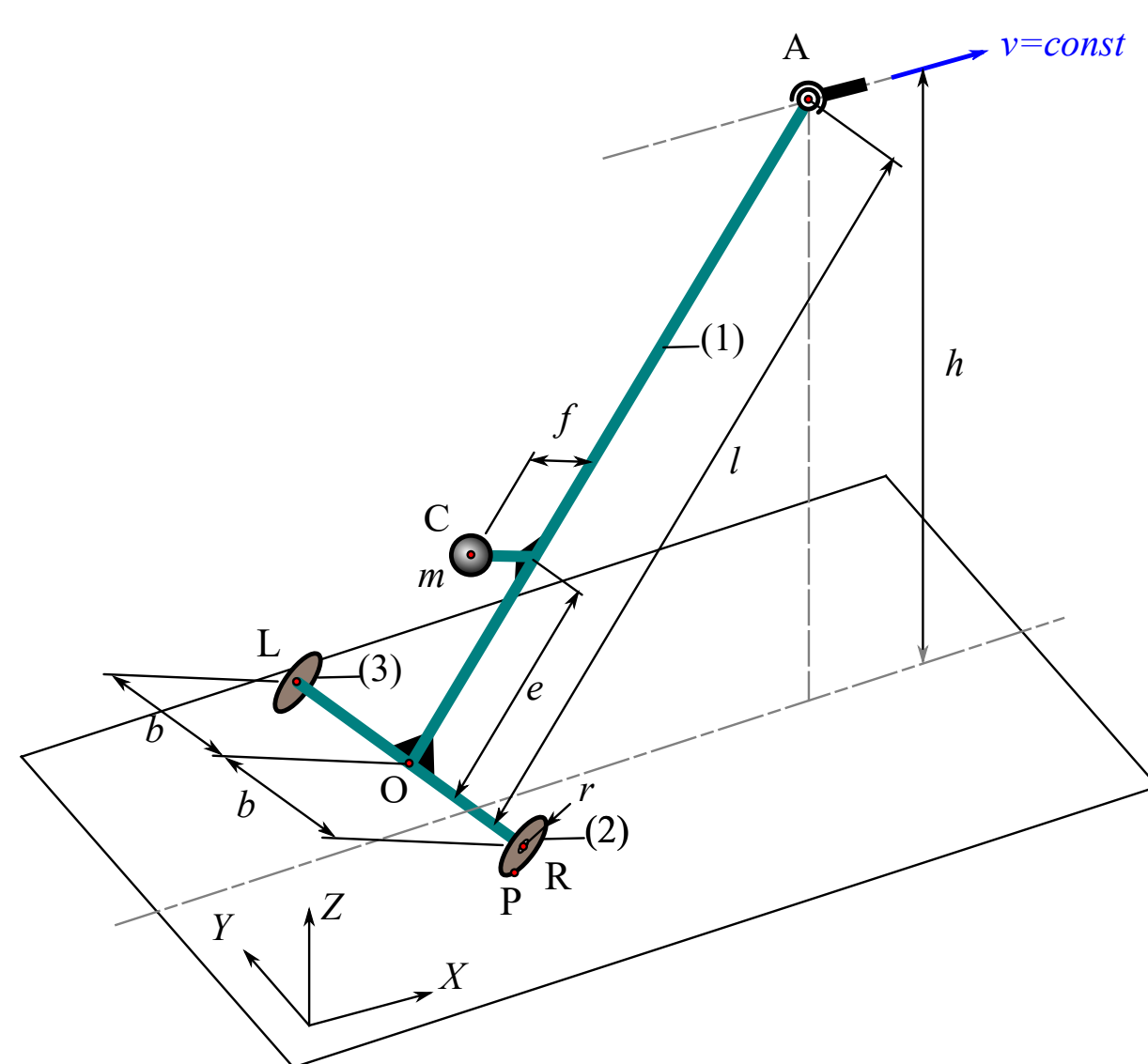


Figure 1: The simplified mechanical model of a towed suitcase.

3. Equations of motion

There are four different motion states: both wheels contacting the ground, one wheel (right or left wheel) on the ground, none of the wheels on the ground, see Fig. 2. The different states correspond to 1, 3 and 6 degrees-of-freedom mechanical models, respectively. The equations of motion can be derived from the Routh-Voss-equation. When both wheels are on the ground, the suitcase makes in-plane motion and the equations of motion can be easily derived. Having only one wheel on the ground is the most intricate case, since the motion is spatial, while the kinematic constraint of rolling has to be considered. If none of the wheels have contact with the ground, the motion is spatial but the system simplifies to a spatial mathematical pendulum. Also, the forces acting on the body have to be taken into consideration.

To make switches between the motion states, a complex impact model is sufficient since classical rigid body impact models could not be used to describe the practically observed transitions between the different states of motion. To do so, a spring and a damping is placed to the impacting wheel. Since the dynamic of the impact is much faster than the change in the body's position, the simulations of the impact can be carried out in another time scale. This method is an established custom for such multi-scale systems. The different impact models (impact on the left or the right or on both wheels) correspond to 2, 2 and 4 degrees-of-freedom mechanical models, respectively.

4. Simulations

The governing equations and the impact model were implemented in computer code. As all motion states had different number of degrees of freedom, the coupling between them is extremely difficult to deal with. The motion states and the possible ways to couple between them can be seen in Fig. 2.

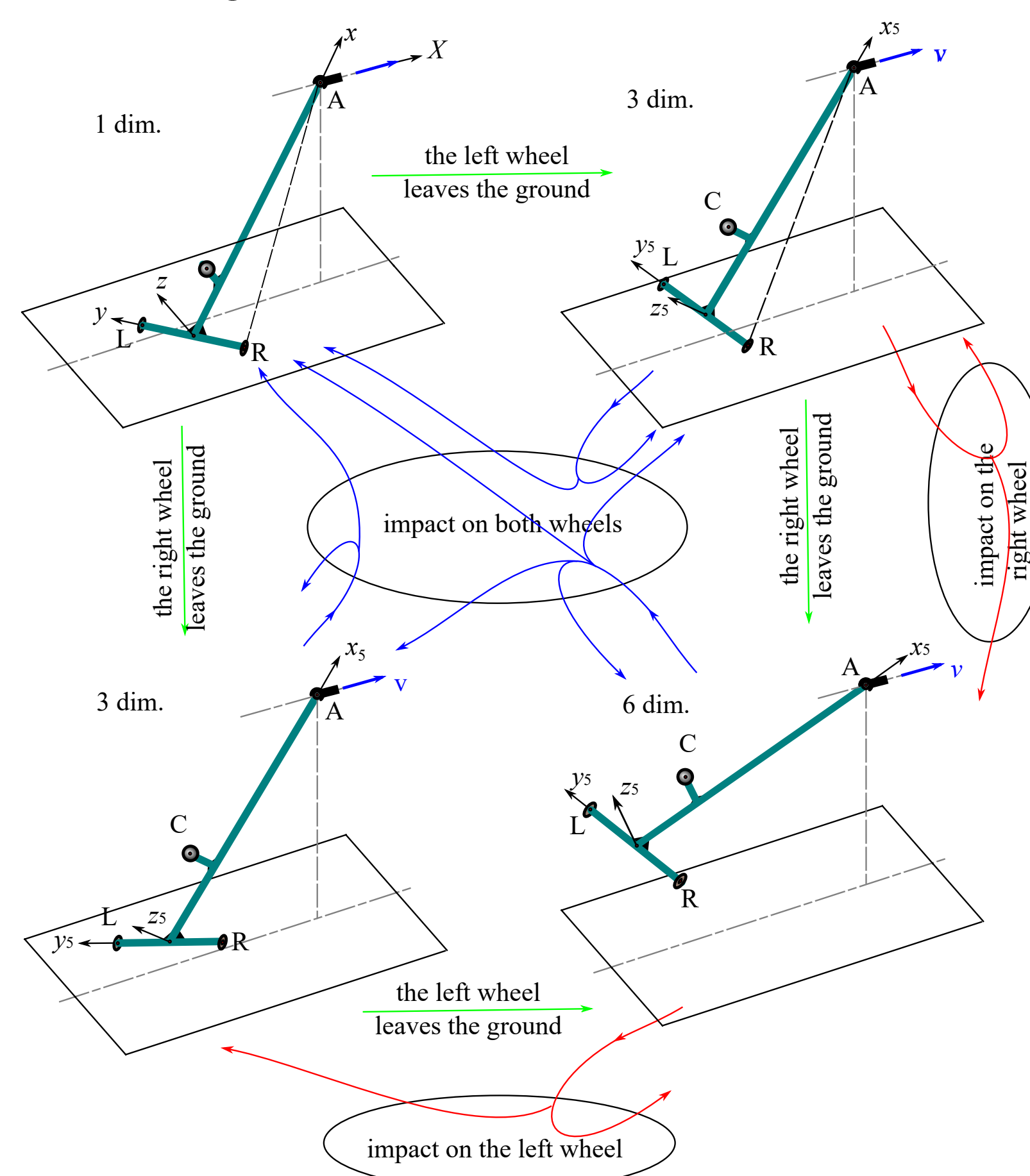


Figure 2: The motion states and the possible ways to couple between them.

Because of the difficulties in the coupling between the motion states, we consider a motion to be stable, if it results in the state in which both wheels are on the ground. Otherwise, we are talking about an unstable motion. Simulations were run for different initial conditions (different β values, where β is the rotational angle around the axis between points A and the centre point of a wheel) and parameters (e.g. towing velocity v). One can draw the basis of attraction of the rectilinear motion by means of these simulations. In Fig. 3, the results of a simplified mechanical model are shown, where the fourth motion state (no wheels on ground) was not considered.

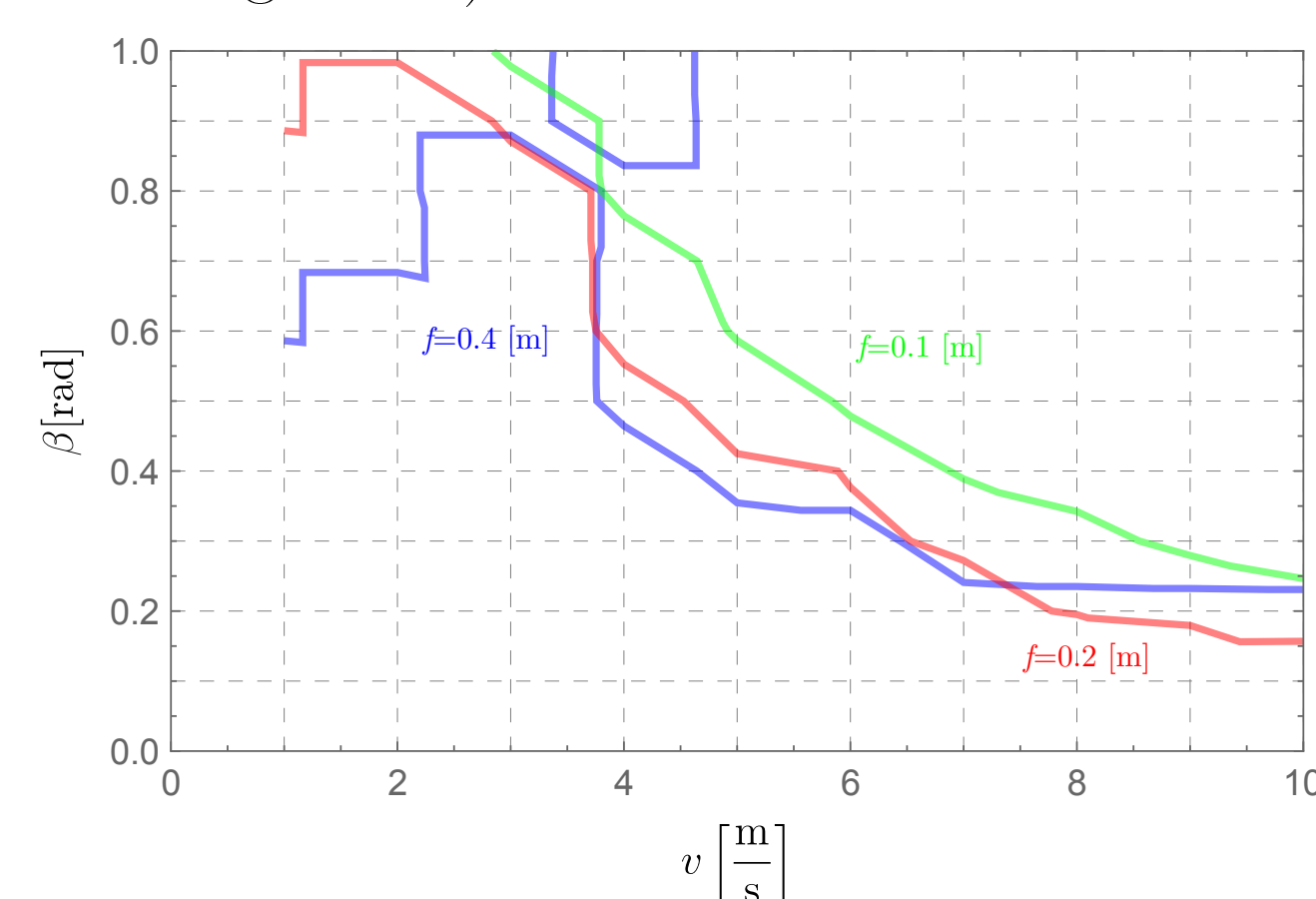


Figure 3: The basis of attraction in such a mechanical model, where the fourth motion state (no wheels on the ground) is not considered. The parameter values are the following: $l = 1$ m, $h = 0.6$ m, $b = 0.3$ m, $m = 10$ kg. Different colours represent different basis of attraction considering different f values.

One can observe the attractive and the repelling regions, i.e. below and above a specific line, respectively. Other figures could be depicted considering different e , b , h or l values. If the value of b or l is increased, the attractive region grows as well. In case of increasing e , f or h , the repelling region grows. As a curiosity, it is worth mentioning that while varying the value of f , there exists such velocities that for specific initial rotational angles, the motion tends to the straight rolling, then while in-

creasing the angle, the motion becomes unstable, then again stable. Such behaviour was observed during the measurements, as well. One can see a snapshot of the simulation in the left panel of Fig. 4.

5. Measurements

Two types of measurement were carried out in order to validate the numerical simulations. While „classical” measurements, the body was placed on a treadmill, so it was towed with its A point fixed with a ball-joint. Since we could not perturb the system identically with every measurement, we could only determine the stability on statistical basis. The measurement setup can be seen in the right panel of Fig. 4.

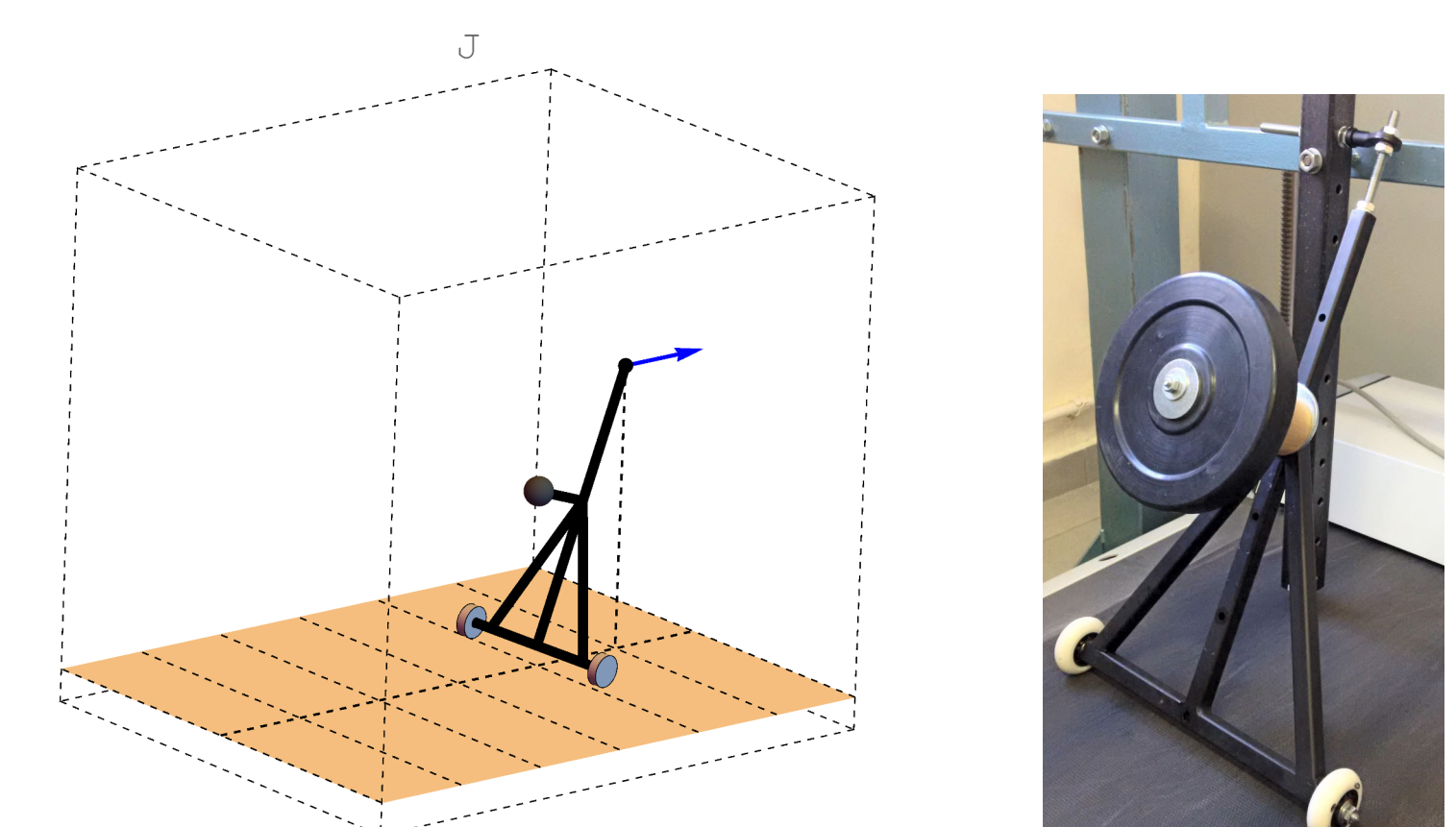


Figure 4: A snapshot of the simulation (left) and the measurement setup (right).

Motion capturing algorithm was also applied in the measurements with 4 markers on the body (on points A, C, R and L). Snapshots about the measurement is shown in Fig. 5, where the rocking motion can be also observed. Evaluating these measurements, we can gain important information about the level of perturbations. Therefore, a more sophisticated identification of the basis of attraction can be carried out.

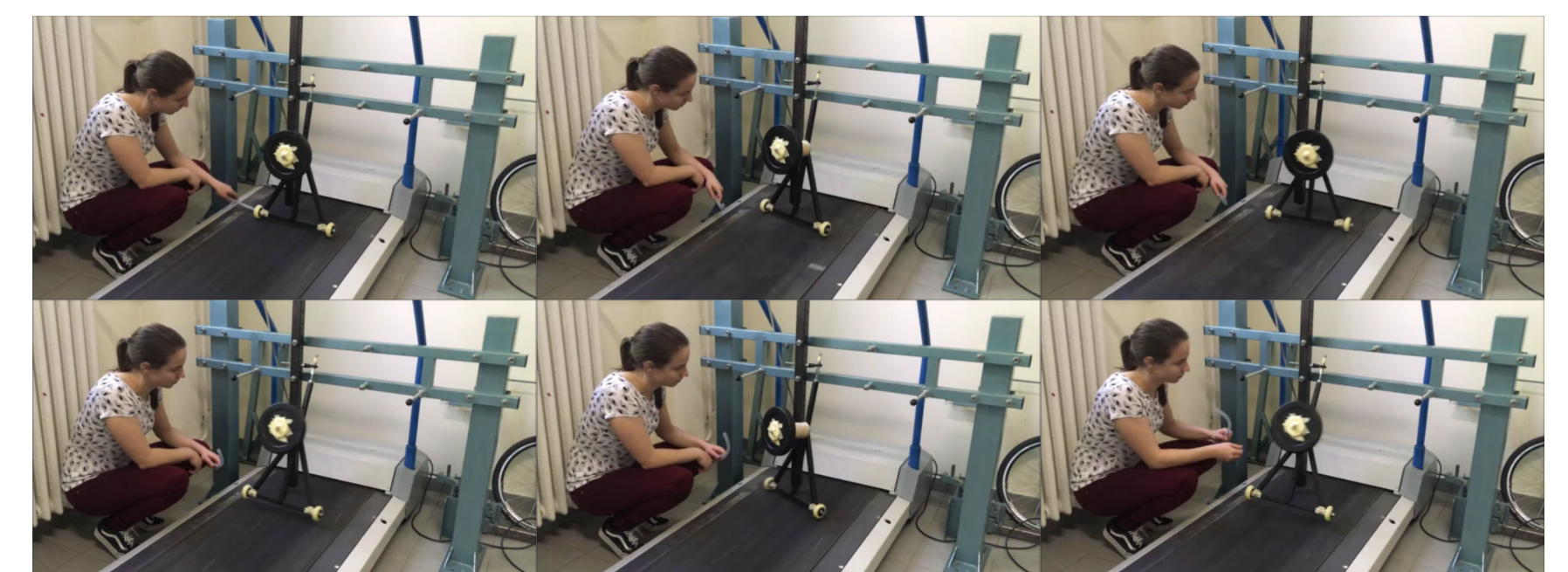


Figure 5: Example from a „motion capturing” measurement - unstable motion.

6. Conclusions

A simplified mechanical model of rocking suitcases was constructed and the equations and motion of the different motion states were derived. The numerical investigation of these equation can result important information about the non-linear behaviour of the suitcase. Experimental rig is also ready to validate these results and provide basic rules that can help the design of suitcases having better stability properties.

Acknowledgements

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