

Chaos in Hamiltonian Systems With Parameter Drift

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Abstract

Based on the example of a paradigmatic low-dimensional Hamiltonian system subjected to different scenarios of parameter drifts of non-negligible rates, we show that the dynamics of such systems can best be understood by following ensembles of initial conditions corresponding to tori of the initial system. When such ensembles are followed, torus-like objects, called snapshot tori are obtained which change their location and shape. In their center one finds a time-dependent, snapshot elliptic orbit. After some time, many of the tori break up and spread over large regions of the phase space, however, one may find some smaller tori, which remain closed curves throughout the whole scenario. We also show that the cause of torus break-up is the collision with a snapshot hyperbolic orbit and the surrounding chaotic sea, which forces the ensemble to adopt chaotic properties. Within this chaotic sea we demonstrate the existence of a snapshot horseshoe structure and a snapshot saddle. An easily visualizable condition for torus break-up is found in relation to a specific snapshot stable manifold. The average distance of nearby pairs of points initiated on an original torus at first hardly changes in time, but crosses over into an exponential growth when the snapshot torus breaks up. This new phase can be characterized by a novel type of finite time Lyapunov exponent which depends both on the torus and on the scenario followed. Tori not broken up are shown to be the analogs of coherent vortices in fluid flows of arbitrary time dependence, and the condition for break-up can also be demonstrated by the so-called PRA method.

References

1. D. JÁNOSI AND T. TÉL. Chaos In Hamiltonian Systems Subjected To Parameter Drift. Chaos 29, 121105 (2019).