

# Polarized baryon production in exact hydrodynamical solutions

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# Aims

- QGP: hot, expanding perfect fluid
- Many known exact hydrodynamical solutions
- At freeze-out baryons get polarized (assuming local thermal equilibrium also for spin degrees of freedom)
- Measurement of this polarization: recent interest
- Polarization could be calculated in exact analytical hydrodynamical solutions

# Known relativistic hydrodynamical solutions

- Many numerical simulations
- Importance of analytic solutions: connection between the initial and final state
- Landau–Khalatnikov solution (1+1D):  
L. D. Landau, Ser. Fiz. 17, 51 (1953).  
I. M. Khalatnikov, Zhur. Eksp. Teor. Fiz. 27, 529 (1954).
- Hwa–Bjorken solution (1+1D):  
R. C. Hwa, Phys. Rev. D 10, 2260 (1974).  
J. D. Bjorken, Phys. Rev. D 27, 140 (1983).
- Hubble flow (1+3D)  
T. Csörgő, L. P. Csernai, Y. Hama and T. Kodama,  
Acta Phys. Hung. A 21 (2004) 73
- Rotating and accelerating expanding solution (1+3D)  
M. I. Nagy, Phys. Rev. C 83, 054901 (2011)

# Investigated solutions

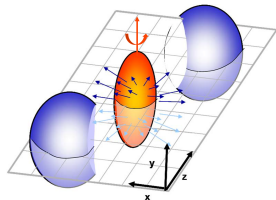
- Hubble flow (1+3D)

T. Csörgő, L. P. Csernai, Y. Hama  
and T. Kodama,

Acta Phys. Hung. A 21 (2004) 73

$$u^\mu = \frac{x^\mu}{\tau}, \quad n = n_0 \left( \frac{\tau_0}{\tau} \right)^3 \mathcal{V}(S),$$

$$T = T_0 \left( \frac{\tau_0}{\tau} \right)^{\frac{3}{\kappa}} \frac{1}{\mathcal{V}(S)} \quad (1)$$



- Accelerating expanding solution

$$\mathbf{v} = \frac{2t\mathbf{r}}{t^2 + r^2 + \rho^2}, \quad n = n_0 \left( \frac{T}{T_0} \right)^3, \quad (2)$$

$$T = \frac{T_0 \tau_0^2}{\sqrt{(t^2 - r^2 + \rho^2)^2 + 4\rho^2 r^2}}.$$

Generalization for rotating expansion:

$$\mathbf{v} = \frac{2t\mathbf{r} + \mathbf{B} \times \mathbf{r}}{t^2 + r^2 + \rho^2}, \quad n = n_0 \left( \frac{T}{T_0} \right)^3, \quad (3)$$

$$T = \frac{T_0 \tau_0^2}{\sqrt{(t^2 - r^2 + \rho^2)^2 + 4\rho^2 r^2 - (\mathbf{B} \times \mathbf{r})^2}}.$$

# Polarization

- At local thermodynamical equilibrium particles get a net polarization proportional to the vorticity of the inverse temperature field

F. Becattini, V. Chandra, L. Del Zanna and E. Grossi,  
Annals Phys. 338, 32 (2013)

- Measurements in STAR: polarization of  $\Lambda$  baryons  
→ analytical solutions are also interesting

# Single-particle distribution

- Single-particle distribution: Fermi–Dirac distribution

$$f(x, p) \propto \frac{1}{\exp \left\{ \beta_\mu p^\mu - \frac{\mu(x)}{T(x)} \right\} + 1}, \quad (4)$$

where  $\beta^\mu = \frac{u^\mu}{T(x)}$  is the so-called inverse temperature-field.

- If the exponent is big, we get back the Maxwell–Boltzmann distribution:

$$f(x^\mu, p^\mu) = \frac{n}{4\pi T^3} \exp \left( - p_\mu \beta^\mu \right). \quad (5)$$

# Single particle momentum distribution and polarization

- Formula for the polarization four-vector:

$$\langle S(x, p) \rangle^\mu = \frac{1}{8m} \varepsilon^{\mu\nu\rho\sigma} (1 - f(x, p)) p_\sigma \partial_\nu \beta_\rho \quad (6)$$

and integrating it over the freeze-out hypersurface, we get

$$\langle S(p) \rangle^\mu = \frac{\int d^3\Sigma_\nu p^\nu f(x, p) \langle S(x, p) \rangle^\mu}{\int d^3\Sigma_\nu p^\nu f(x, p)} \quad (7)$$

F. Becattini and I. Karpenko, Phys. Rev. Lett. 120, no. 1, 012302 (2018)

- Cooper–Frye formula for the invariant momentum spectrum:

$$E \frac{dN}{d^3\mathbf{p}} = \int d^3\Sigma_\nu p^\nu f(x, p), \quad (8)$$

where  $\Sigma$  is the space-like 3-dimensional freeze-out hypersurface.

F. Cooper and G. Frye, Phys. Rev. D 10, 186 (1974).

# Saddle-point integration method

- Calculate the following integration:

$$\begin{aligned}\int g(\mathbf{r})h(\mathbf{r})d^3\mathbf{r} &\approx g(\mathbf{R}_0)h(\mathbf{R}_0) \int d^3\mathbf{r} \exp\left\{-\frac{1}{2}(\mathbf{r} - \mathbf{R}_0)\mathbf{M}(\mathbf{r} - \mathbf{R}_0)\right\} = \\ &= g(\mathbf{R}_0)h(\mathbf{R}_0)\sqrt{\frac{(2\pi)^3}{\det \mathbf{M}}}.\end{aligned}\tag{9}$$

$\mathbf{R}_0$ : saddle point

$\mathbf{M}$ : second derivative matrix

# Polarization four-vector

- Using the saddle-point method and with the approximation:

$$f(x, p) \propto \frac{1}{\exp \left\{ \beta_\mu p^\mu - \frac{\mu(x)}{T(x)} \right\} + 1} \ll 1, \quad (10)$$

we can write

$$\langle S(x, p) \rangle^\mu = \frac{1}{8m} \varepsilon^{\mu\nu\rho\sigma} p_\sigma \partial_\nu \beta_\rho \quad (11)$$

and we get

$$\langle S(p) \rangle^\mu = \frac{\int d^3\Sigma_\nu p^\nu f(x, p) \langle S(x, p) \rangle^\mu}{\int d^3\Sigma_\nu p^\nu f(x, p)} \approx \frac{1}{8m} \varepsilon^{\mu\nu\rho\sigma} p_\sigma \partial_\nu \beta_\rho \Big|_{\mathbf{r}=\mathbf{R}_0}. \quad (12)$$

# Hubble flow

Interest: polarization four-vector and invariant momentum spectrum.

- Velocity, particle density and temperature fields:

$$u^\mu = \frac{x^\mu}{\tau}, \quad n = n_0 \left( \frac{\tau_0}{\tau} \right)^3, \quad T = T_0 \left( \frac{\tau_0}{\tau} \right)^{\frac{3}{\kappa}}. \quad (13)$$

- $\tau = \tau_0 = \text{const.}$  freeze-out hypersurface.
- Saddle-point:  $\mathbf{R}_0 = \frac{\tau_0}{m} \mathbf{p}$
- Polarization

$$\langle S(p) \rangle^0 = 0, \quad \langle S(p) \rangle^i = 0 \quad \Rightarrow \quad \langle S(p) \rangle^\mu = 0 \quad (14)$$

- constant invariant momentum spectrum

# Rotating and expanding solution

Fields:

$$\mathbf{v} = \frac{2t\mathbf{r} + \mathbf{B} \times \mathbf{r}}{t^2 + r^2 + \rho^2}, \quad n = n_0 \left( \frac{T}{T_0} \right)^3, \quad T = \frac{T_0 \tau_0^2}{\sqrt{(t^2 - r^2 + \rho^2)^2 + 4\rho^2 r^2 - (\mathbf{B} \times \mathbf{r})^2}}. \quad (15)$$

- $\tau = \tau_0 = \text{const.}$  freeze-out hypersurface is chosen.
- The saddle-point:  $\mathbf{R}_0 = \alpha \mathbf{p} + \beta (\mathbf{p} \times \mathbf{B})$ ,

$$\alpha = 2\beta \sqrt{\frac{\tau_0^2 + \beta^2 D^2}{1 - 4\beta^2 p^2}} \quad \text{and} \quad \beta = \frac{\varepsilon - m}{2p^2}. \quad (16)$$

- Momentum spectrum:

$$\varepsilon \frac{dN}{d^3\mathbf{p}} \propto \sqrt{\frac{\pi^3 T_0^3 \tau_0^3}{32p(m + \varepsilon)}} \exp\left(-\frac{E_{\text{eff}}}{T_0}\right), \quad (17)$$

where

$$E_{\text{eff}} = m + \frac{\rho^2 \varepsilon}{\tau_0^2} + \frac{p^2 B^2 - (\mathbf{pB})^2}{4p^2 \tau_0^2} (\varepsilon - m) \quad (18)$$

# Main topic: polarization in rotating and expanding solution

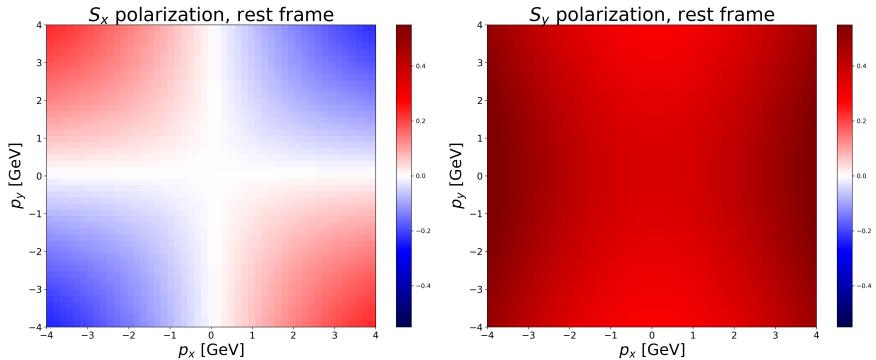
- Polarization four-vector:

$$\langle S(p) \rangle^\mu = \frac{1}{8m} \begin{pmatrix} \mathbf{pB} \\ \varepsilon \mathbf{B} \end{pmatrix} \quad (19)$$

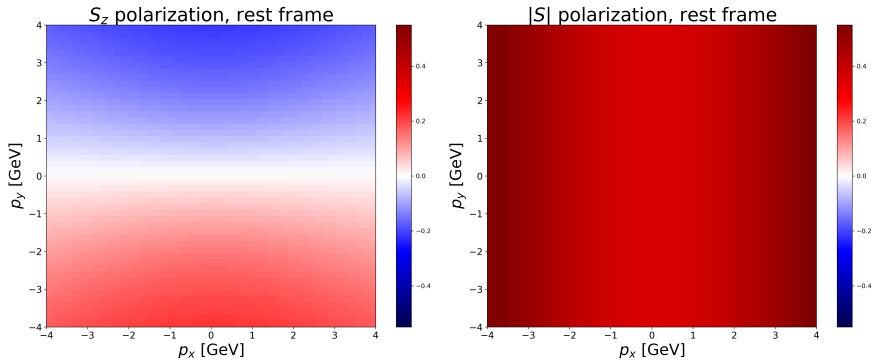
- In the rest frame we get:

$$\langle S(p) \rangle_{\text{rest frame}}^\mu = \frac{1}{8m} \begin{pmatrix} 0 \\ (m - \varepsilon) \frac{\mathbf{p}(\mathbf{pB})}{p^2} + \varepsilon \mathbf{B} \end{pmatrix}. \quad (20)$$

# Polarization vector figures: rotating and expanding solution



# Polarization vector figures: rotating and expanding solution



## Further investigations

- Hubble solution

T. Csörgő, L. P. Csernai, Y. Hama and T. Kodama, Acta Phys. Hung. A 21 (2004) 73

$$u^\mu = \frac{x^\mu}{\tau}, \quad n = n_0 \left( \frac{\tau_0}{\tau} \right)^3 \mathcal{V}(S), \quad T = T_0 \left( \frac{\tau_0}{\tau} \right)^{\frac{3}{\kappa}} \frac{1}{\mathcal{V}(S)}, \quad (21)$$

where

$$\mathcal{V}(S) = 1 + bS, \quad S(\tau) = \frac{r_x^2}{\dot{X}_0^2 \tau^2} + \frac{r_y^2}{\dot{Y}_0^2 \tau^2} + \frac{r_z^2}{\dot{Z}_0^2 \tau^2}. \quad (22)$$

- Rotating Hubble-like solution

M. I. Nagy, Phys. Rev. C83, 054901 (2011)

$$\mathbf{v} = \frac{\mathbf{r} + \mathbf{E}t + \mathbf{B} \times \mathbf{r}}{t + (\mathbf{E}\mathbf{r})}, \quad n = n_0 \left( \frac{T}{T_0} \right)^\kappa, \quad (23)$$

$$T = \frac{T_0 \tau_0}{\sqrt{t^2 - r^2 + (\mathbf{B}\mathbf{r})^2 - B^2 r^2 - E^2 t^2 + (\mathbf{E}\mathbf{r})^2}}$$

# Conclusions

- Polarization four-vector and the invariant single-particle momentum distribution
  - Spherical symmetric Hubble flow
  - Rotating and expanding solution
- Many more solutions could be investigated
- Disentangle the effect of acceleration, rotation and temperature gradient

Thank you for your attention!

# Hubble flow

- Momentum spectrum:

$$\varepsilon \frac{dN}{d^3\mathbf{p}} = \frac{n_0}{4} \sqrt{\frac{\pi \tau_0^3}{m T_0^3}} \exp\left(-\frac{\tau_0 m}{T_0}\right) \quad (24)$$

- For the polarization four-vector  $\partial_\nu \beta_\rho$  is needed:

$$\partial_\nu \beta_\rho = \frac{g_{\nu\rho}}{\sqrt{\tau_0^2 + r^2} T_0} + \frac{x_\nu x_\rho}{(\tau_0^2 + r^2)^{3/2} T_0} \quad (25)$$

- Using that  $\varepsilon^{\mu\nu\rho\sigma}$  is antisymmetric and  $\partial_\nu \beta_\rho$  is symmetric we get:

$$\langle S(p) \rangle^0 = 0, \quad \langle S(p) \rangle^i = 0 \quad \Rightarrow \quad \langle S(p) \rangle^\mu = 0 \quad (26)$$