

CLIQUE NUMBER OF XOR PRODUCTS OF KNESER GRAPHS

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ABSTRACT. In this article we investigate a problem in graph theory, which has an equivalent reformulation in extremal set theory similar to the problems researched in [13] by Gyula O.H. Katona, who proposed our problem as well. In the graph theoretic form we examine the clique number of the Xor product of two isomorphic $KG(N, k)$ Kneser graphs. Denote this number with $f(k, N)$. We give lower and upper bounds on $f(k, N)$, and we solve the problem up to a constant deviation depending only on k , and find the exact value for $f(2, N)$ if N is large enough. We also compute that $f(k, k^2)$ is asymptotically equivalent to k^2 .

1. INTRODUCTION

In the abstract we briefly stated the problem in the graph theoretic form, here we introduce the definitions that are necessary to understand the statement.

Definition 1.1 (Kneser graphs). A Kneser graph $KG(N, k)$ is a graph defined as follows. The vertices correspond to the k element subsets of a given N element set, and two vertices are connected if and only if the corresponding subsets are disjoint.

The following graph product was studied in [17].

Definition 1.2 (Xor product). Given two graphs, G and H , let us denote by $V(G)$, $V(H)$ and $E(G)$, $E(H)$ their vertex and edge sets respectively. Define the Xor product, denoted by $G \cdot H$, as the graph with the vertex set $V = V(G) \times V(H)$ (where $V(G) \times V(H)$ denotes the Cartesian product of the vertex sets), and two vertices (g, h) and (g', h') are connected if and only if among the statements $gg' \in E(G)$ and $hh' \in E(H)$ exactly one occurs.

Definition 1.3 (Clique number). Let $G = (V, E)$ be a simple graph. A subset $H \subset V$ is called a clique, if any two vertices in H are connected by an edge in G . The clique number of G is simply the size of a maximal clique, denoted by $\omega(G)$.

So now we have everything to state the main problem of this article.

Problem 1.4. We want to find the value of $\omega(KG(N, k) \cdot KG(N, k))$.

As the exact value for all pairs (k, N) is beyond our reach, we focus on giving lower and upper bounds, determining the asymptotic behaviour of these functions, and calculating the exact value for a large family of pairs (k, N) .

Similar questions have been studied about the independence number of different, more conventional (Cartesian, Tensor and Strong) products of the Kneser graph in [5]. The clique number in these graphs for any conventional graph product can be determined

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