

Covers, orientations and factors

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Abstract

Given a graph G with only even degrees, let $\varepsilon(G)$ denote the number of Eulerian orientations, and let $h(G)$ denote the number of half graphs, that is, subgraphs F such that $d_F(v) = d_G(v)/2$ for each vertex v . Recently, Borbényi and Csikvári proved that $\varepsilon(G) \geq h(G)$ holds true for all Eulerian graphs, with equality if and only if G is bipartite. In this paper we give a simple new proof of this fact, and we give identities and inequalities for the number of Eulerian orientations and half graphs of a 2-cover of a graph G .

Mathematics Subject Classifications: 05C30, 05C70, 05C76, 05C45

1 Introduction

In this paper we study the number of orientations and factors of Eulerian graphs. Recall that a graph G is Eulerian if every vertex of G has even degree. In the literature it is

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