

Optical flow-based vertical angular rate fault detection on UAVs

Gabor Jevuczo* Peter Bauer*

** Systems and Control Laboratory, Institute for Computer Science and Control (HUN-REN SZTAKI), HUN-REN, Budapest, Hungary
(e-mail: bauer.peter@sztaki.hun-ren.hu)*

Abstract:

This paper discusses optical flow-based vertical angular rate fault detection based-on real flight IMU data and camera image sequences. The flight test platform utilizes a gimbal stabilized downward looking camera that is why only the angular rate around the inertial vertical axis can be estimated. Improvements relative to the previous work of the authors are the application of forward-backward optical flow to filter outliers, the direct detection of angular rate faults without the need for Euler angles or GPS velocities and tuning and verification on real images and flight data. After presenting the optical flow equations and the method for image-based vertical angular rate estimation the flight test scenarios and video processing steps are introduced. Four different artificial errors are added to the IMU measurements and the fault detection applies residual thresholding and up-down counters. Tuning is based on false alarm and missed detection rates and the application of receiver operating characteristics. Finally, the estimation results and possible further improvements are presented.

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Keywords: angular rate fault detection, real flight video, optical flow, UAV

1. INTRODUCTION

The fault detection of aerial vehicle onboard systems is a crucial task according to Goupil et al. (2015) and Heredia et al. (2005). With the widespread use of UAVs (Unmanned Aerial Vehicles) and according to the new EU regulations European Union (2019) redundancy and tolerance of single faults is mandatory also for small UAVs. Besides the application of multiple IMU and GPS sensors in case of UAVs also the on-board camera system can be considered as a possible source of motion information (Kehoe et al. (2006), Simlinger and Ducard (2019), Hamaoui (2015), Santamaria-Navarro et al. (2015), Hiba et al. (2021) and Hiba et al. (2023)). As Song, He et al. (2017) points out the angular rate sensors are core parts of the navigation so the authors previous works (Kun and Bauer (2021) and Bauer and Kun (2022)) have dealt with its mono camera-based estimation from optical flow (OF) and OF-based fault detection in case of IMU failures. OF is widely applied for angular rate and/or velocity estimation Fasano et al. (2013), Hamaoui (2015), Santamaria-Navarro et al. (2015), Fayad (2014), Xiang et al. (2021) and Gebert et al. (2003) and so navigation and control of systems. Kehoe et al. (2006) presents a nonlinear optimization-based solution, Hamaoui (2015) transforms the problem with the differential epipolar constraint but again ends with a nonlinear optimization. Gebert et al. (2003) considers a narrow field of view (FOV) camera (constant range for every feature) and removes the velocity effects to have a linear equation for the angular rate. Santamaria-Navarro et al. (2015) and Fayad (2014) apply downward looking cameras fixed to the drone to estimate vehicle states facing the problem of similar OF effects of vehicle pitch/roll and forward/side

motion. Xiang et al. (2021) applies three OF sensors to solve this problem. Kun and Bauer (2021) considered an aircraft with forward looking camera placed at the center of gravity (CG) and derived a two step linear solution for the nonlinear problem knowing that the side and up velocity components are small. It was compared to the nonlinear solutions published in literature (Kehoe et al. (2006)) based-on simulated feature point data. Bauer and Kun (2022) further developed the method with a camera away from the CG and unaligned with body system. The same two step linear solution was applied considering synthetic image (from UNREAL Carla simulation) and IMU sensory data. Fault detection of the angular rate sensors was presented assuming known velocity and Euler angles and considering a wide range of possible sensor faults based-on Heredia et al. (2005). As the Euler angles can be corrupted in case of angular rate fault this method is not safe for real life application.

The main goal of the current work is to further develop and apply the previous results on real camera images and IMU measurements. Due to the limited resources a multicopter with a gimbal stabilized downward looking camera utilized in another project (see Bauer et al. (2023)) was applied for data collection. In this system there is no pitch/roll and forward/side motion coupling due to the gimbal stabilization but only the angular rate around the inertial vertical axis (vertical angular rate) can be observed from the OF. Improvements relative to the previous work include the application of forward-backward optical flow to remove outliers and the consideration of special OF equations for the vertical angular rate.

The OF-based estimates are applied in angular rate fault detection superimposing the possible faults from Heredia et al. (2005) to the real IMU measurements. Contrary to Bauer and Kun (2022) a direct comparison is applied on the estimated and measured angular rates cancelling the need for any additional and possibly corrupted measured data. The structure of the paper is as follows.

Section 2 introduces the optical flow equations and the method of vertical angular rate estimation. Section 3 briefly describes the test flight trajectories and the workflow of video processing. Section 4 introduces the sensor errors, the method of fault detection and its results. Finally, Section 5 concludes the paper.

2. OPTICAL FLOW EQUATIONS AND VERTICAL ANGULAR RATE ESTIMATION

The considered coordinate systems (see Fig. 1) are the North-East-Down (NED) earth system, the body system (X_b, Y_b, Z_b) fixed to the center of gravity of the drone and the camera system (X_c, Y_c, Z_c) fixed to the camera sensor. During flight the camera is vertically stabilized by a gimbal. Position of the camera system in body is denoted by (Δ^B) . The body angular rate, body velocity and the camera position vectors are transformed to camera system determined by the Z-Y-X Euler angle rotations. The rotation matrix between the two systems is defined by T_{CB} resulting $\omega^C = T_{CB}\omega^B = [p^C \ q^C \ r^C]^T$, $V^C = T_{CB}V^B = [u^C \ v^C \ w^C]^T$ and $\Delta^C = T_{CB}\Delta^B = [x^C \ y^C \ z^C]^T$.

Furthermore the following notations are used. F_i is the i -th feature point. (η_i^C) is the i -th feature position vector in the camera system. (μ_i, ν_i) are image coordinates of the i -th feature point at $Z=f$ focal length in pixel (px) dimension, see Fig. 1.

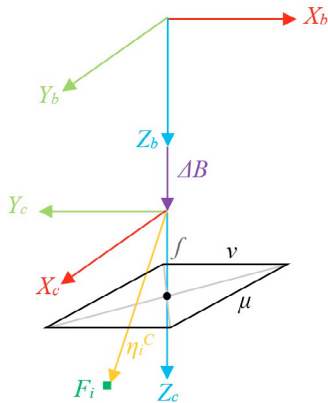


Fig. 1. The coordinate systems

The derivation of the optic flow equations in the camera system is based on the work presented in Bauer and Kun (2022).

$$\begin{aligned} \dot{\mu}_i = & r^C \left(\nu_i + \frac{f y^C}{\eta_{3i}} \right) - q^C \left(f + \frac{\mu_i^2}{f} + \frac{f z^C}{\eta_{3i}} + \frac{\mu_i x^C}{\eta_{3i}} \right) \\ & + p^C \left(\frac{\mu_i y^C}{\eta_{3i}} + \frac{\mu_i \nu_i}{f} \right) + \frac{\mu_i w^C}{\eta_{3i}} - \frac{f u^C}{\eta_{3i}} \\ \dot{\nu}_i = & p^C \left(f + \frac{\nu_i^2}{f} + \frac{f z^C}{\eta_{3i}} + \frac{\nu_i y^C}{\eta_{3i}} \right) - r^C \left(\mu_i + \frac{f x^C}{\eta_{3i}} \right) \\ & - q^C \left(\frac{\nu_i x^C}{\eta_{3i}} + \frac{\mu_i \nu_i}{f} \right) - \frac{f v^C}{\eta_{3i}} + \frac{\nu_i w^C}{\eta_{3i}} \end{aligned} \quad (1)$$

Where $\eta_{3i} = \eta_i^C(3)$. Considering that the camera is mounted along the Z axis the value of x^C and y^C is approximately zero. Furthermore we assume that during the flight $w^C = 0$ because the drone holds a constant altitude and the camera is aligned with the NED down (inertial vertical) axis by the gimbal. Respectively we assume that besides r^C all the other angular rate components p^C and q^C are close to zero. With these approximations (1) can be simplified as:

$$\dot{\mu}_i = \nu_i r^C - \frac{f u^C}{\eta_{3i}} \quad (2)$$

$$\dot{\nu}_i = -\mu_i r^C - \frac{f v^C}{\eta_{3i}} \quad (3)$$

The unknowns are r^C, u^C, v^C, η_{3i} . Considering an instantaneous constant ratio between the velocities $u^C = k v^C$ results in:

$$\frac{f u^C}{\eta_{3i}} = k \frac{f v^C}{\eta_{3i}} \quad (4)$$

Expressing $\left(\frac{f v^C}{\eta_{3i}}\right)$ from (3) and substituting into (4) the result can be substituted into (2) resulting in:

$$\dot{\mu}_i = \nu_i r^C + k \mu_i r^C + k \dot{\nu}_i \quad (5)$$

In (5) the only unknowns are r^C and k so the dependence on the feature dependent η_{3i} was eliminated. (5) is a bilinear equation therefore instead of the least squares method still an iterative solution is required to get r^C . In the first iteration k is considered as zero and r^C is calculated with least squares method. Then the result is substituted into (5) and k is calculated. Then the calculated k is substituted and a new r^C is calculated and so on. The number of iterations was limited to 100. In the case of onboard implementation a convergence criterion should be applied but in offline calculation all 100 steps were executed since computational time is not critical.

Considering equation (4) $k = \frac{u^C}{v^C}$ can lead to singularity if $v^C = 0$ or close to zero. Fig. 2 shows that a zero crossing of v^C causes spikes in k (calculated from the sensor data of Flight1 (flight details are presented in the next Section 3)).

Detection of the singularities can be possible analysing the equations. In (3) if v^C equals to zero than $\mu_i r^C + \dot{\nu}_i = 0$ can be written. Substituting this into a rearranged version of (5) shows that both sides of the following equations (for multiple points) should be zero:

$$\dot{\mu}_i - \nu_i r^C = k (\mu_i r^C + \dot{\nu}_i) \quad (6)$$

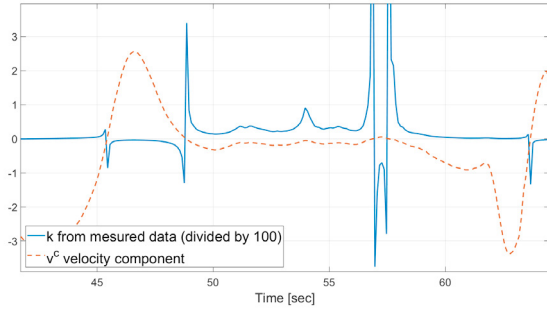


Fig. 2. Value of k calculated from sensor data.

Calculating the Frobenius norm of both sides (Frobenius norm of a zero vector or close to a zero vector should result in a value zero or close to zero) shows that due to the disturbances and noises caused by pixel faults and gimbal angle measurement uncertainties their norms will not be close to zero (see Fig. 3).

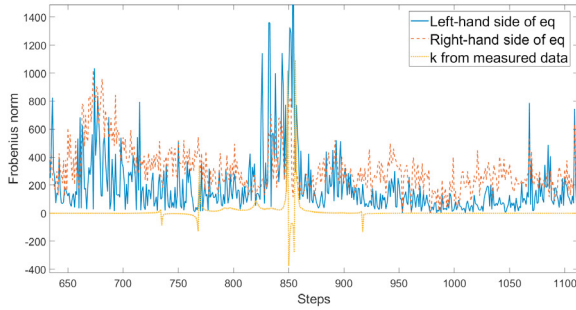


Fig. 3. Frobenius norms of both sides of eq. (6).

However, due to the same disturbances there is no singularity problem in (5). This can be seen in Fig. 4 where k calculated from the measured data (divided by 100) and k from optical flow are plotted together. So finally the detection of the singularity was not necessary.

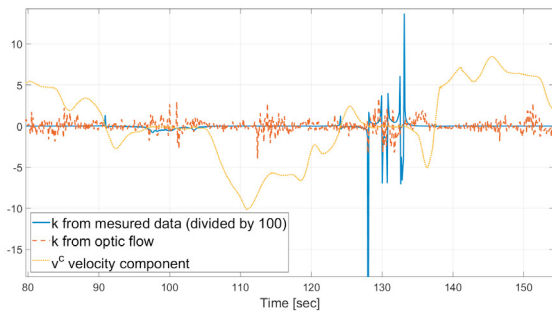


Fig. 4. Calculated k from optical flow and measured data.

3. FLIGHT TEST TRAJECTORIES AND VIDEO PROCESSING

Two flight test trajectories of a DJI M600 multicopter equipped with a downward looking camera (see Bauer et al. (2023)) were considered in this work (see Fig. 5).

The first is a faster ($\sim 6\text{m/s}$) large square route with longer turns ($\sim 6\text{sec}$) having also a moving car in camera field of view (Flight1), while the second is a slower ($\sim 1\text{m/s}$) small square route with shorter turns ($\sim 4\text{sec}$) without car (Flight2).



Fig. 5. Multicopter drone flight tracks above ZalaZONE Smart City (Flight1: blue, Flight2: red)



Fig. 6. Detected feature points.

To calculate optical flow it is necessary to find a proper number of feature points on all consecutive frames. This requires the images to be processed one by one from the video. First *ORB* (Oriented FAST and Rotated BRIEF with 600 maximum number of feature points and 1 image pyramid level) OpenCV function was applied to find the feature points on the first frame (indicated with index t). To find and match these points on the following frame, as a forward OF (index $t+1$), *OpticalFlowPyrLK* (sparse optical flow with iterative Lucas-Kanade method applying pyramids with 9×9 window size, 7 max. level, 10 count and 0.03 epsilon) OpenCV function was used. To eliminate incorrect matches between the tracked points, as a backward OF, *OpticalFlowPyrLK* was applied on the $t+1$ feature points to find them on the first frame.

A threshold was applied based on the Euclidean distance between the points defined by the *ORB* and the points calculated from the backward OF. If the distance between the points exceeds the threshold than those non-matching points are removed. This helped to have a better angular rate estimation. Due to the camera lens a barrel distortion correction was applied on the identified feature points before OF calculation.

As shown in Bauer and Kun (2022) the optical flow velocities can be approximated as first order numerical derivatives (with Δt sample time):

$$\dot{\mu}_i \approx \frac{\mu_i(t+1) - \mu_i(t)}{\Delta t}, \quad \dot{\nu}_i \approx \frac{\nu_i(t+1) - \nu_i(t)}{\Delta t}$$

4. ANGULAR RATE ERRORS AND FAULT DETECTION

The following sensor errors were considered, based on Heredia et al. (2005) where a detailed description of angular rate sensor errors can be found.

- (1) Total sensor failure (sensor gives 0 as its output after the fault occurs)
- (2) Stuck with constant value sensor failure (sensor gives last value as constant output after the fault occurs)
- (3) Additive-type sensor failure (A rad/s added to the measured value)
- (4) Multiplicative-type sensor failure (sensor output is $100+P\%$ of the measured value)

To detect these sensor faults, an up-down counter-based residual thresholding was applied based on the work of Ozdemir et al. (2011).

The DJI M600 drone has three independent angular rate sensors from these sensors 1 and 2 were considered. The sensor faults listed above were generated on the data of sensor 2 and sensor 1 was kept as a reference for an error free sensor. Note that the noise and bias characteristics of the two sensors are different so they never perfectly cover each others output. Sensor outputs were compared to the estimated OF-based angular rate (r_{OF}) resulting in a residual (e) for which, a threshold can be introduced ($Tol1$).

$$e_{1/2} = r_{sensor1/2} - r_{OF}$$

If this residual exceeds the threshold ($Tol1$) than the sensor's counter is incremented with the up-count amount (U), if not, the counter is decremented by the down-count amount (D) considering a 0 lower bound for the counter. For the counter a fault declaration threshold ($Tol2$) can be introduced and if the counter reaches this threshold then a fault can be declared.

To obtain $Tol1$, similarly to Grof and Bauer (2020) the false alarm (FA) rate $P(data > Tol1|no\ error)$ was defined as the probability of the residual exceeding $Tol1$ while there is no error in the signal and the missed detection (MD) rate $P(data < Tol1|error)$ which is the probability of the residual being lower than $Tol1$ while there is an error.

To calculate MD probability (P MD), stuck with constant value fault was generated on the data of sensor 2 with

smaller (Flight1: $11.18^\circ/s$ Flight2: $-9^\circ/s$) and higher (Flight1: $-29.3^\circ/s$ Flight2: $23.45^\circ/s$) error values.

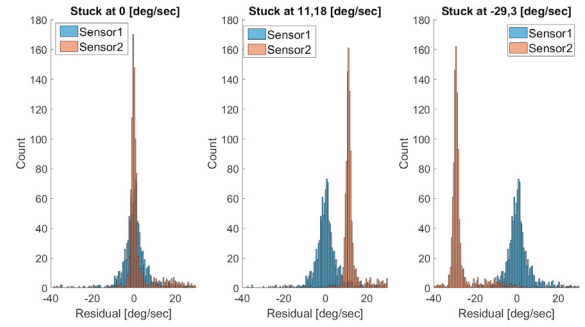


Fig. 7. Angular rate error residual histograms of Flight1 with different sensor errors (sensor 1 is healthy, fault is on sensor 2)

Total sensor failure (zero output value) was not considered as its residual can not be distinguished from the healthy sensor residual as shown in Fig.7. When there is no turning the lack of excitation can prevent the detection of small stuck errors while at the same time sensor noise can give false alarms. To eliminate this $P(data < Tol1|error)$ and $(Pdata > Tol1|no\ error)$ were calculated only when the measured signal exceeds $4^\circ/s$ denoting this threshold as *angularTHS*. This value was obtained experimentally by calculating the curves (like Fig. 8) for multiple *angularTHS* values considering the noise on the residuals.

By counting $(data < Tol1|error)$, $(data > Tol1|no\ error)$ in every time step with multiple $Tol1$ values between $0^\circ/s$ and $34^\circ/s$ for both flights and dividing it by the number of time steps will give the FA and MD probability distributions shown in Fig. 8. The figure shows that by increasing the threshold the FA rate decreases while the MD rate increases.

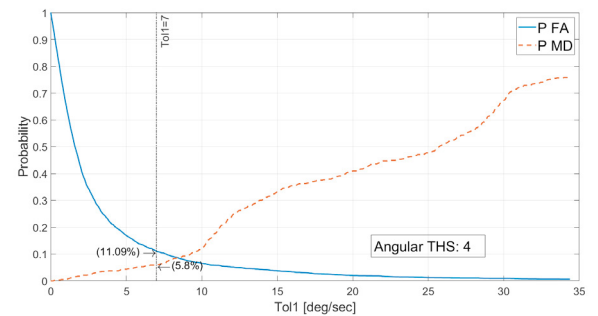


Fig. 8. FA and MD probability distributions

Threshold $Tol1$ selection is a trade-off between FA and MD rate. In our case the FA and MD rate is equal at around $8.37^\circ/s$ $Tol1$ but a value $Tol1 = 7^\circ/s$ at lower MD and a higher FA rate was chosen because with a well selected Up-Down counter the false alarm rate can be lowered.

$Tol2$ was determined with the use of Receiver Operating Characteristic (ROC) curve. A detailed description of the ROC curve can be found in Zou et al. (2007). The ROC curve shows the False Positive rate (FPR) with respect to the True Positive rate (TPR) throughout multiple threshold values (see Fig. 9).

FPR is calculated with error-free sensor data by counting how many times the counter exceeds $Tol2$ on both flights and dividing it by the sum of false positives and true negatives.

To acquire the true positive rate stuck with constant value faults were generated during turns in 25 different times for both flights individually. The TPR can be calculated by counting how many times the counter exceeds $Tol2$ for both flights and divide it by the sum of true positives and false negatives.

Both FPR and TPR was calculated only when the sensor data exceeds $angularTHS$ ($4^\circ/s$). By repeating these calculations for all $Tol2$ values between 0 and 100 the following ROC curve can be constructed (see Fig. 9).

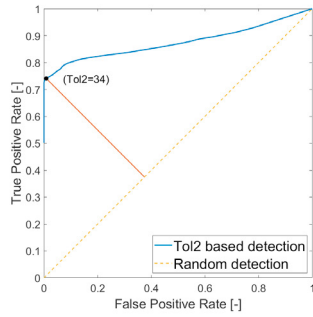


Fig. 9. ROC curve to find optimal $Tol2$.

After the construction of the curve, the optimal value for $Tol2$ was found by the largest perpendicular distance from the diagonal line, thus $Tol2 = 34$ was selected.

4.1 Fault detection results

Table 1. Fault detection results.

Error Type	Flight1 Varying	Flight1 Steady	Flight2 Varying	Flight2 Steady
1	2.17s	5.367s	67.4377s	69.5392s
2	13.795s	5.367s	67.6409s	69.5392s
3	55.677s	59.677s	X	X
4	51.267s	55.267s	67.5392s	69.5392s

Table 1 shows the detection times from the fault activation times in case of successful detection and X marks the missed detections with the previously defined parameters ($Tol1 = 7^\circ/s$ $Tol2 = 34$ $angularTHS = 4^\circ/s$ $U = 1$ $D = 1$).

In Table 1 steady case means that the fault is activated when the angular rate is close to constant and varying case means that the fault is activated before large changes in yaw rate (impersistent and persistent excitation).

In case of additive-type sensor failure this method was able to detect $A = 2.1^\circ/s$ and higher added constant values on Flight1 and with multiplicative-type sensor failure it was able to detect $P = 8\%$ and higher percentages of the measured signal in both flights. The values in Table 1 belong to the smallest value of A and P that could be detected, with higher deviations the detection times should decrease.

The additive-type sensor failure on Flight2 was not detected since the error counter did not reach the $Tol2$ value

(see Fig. 12) within the examined time frame of Flight2 (only a segment of Flight2 was processed) due to the turns short duration and the 10Hz sampling. Similar happens with the other error types on Flight2, the long detection times are caused by the short turns from which only the second can drive the counter above $Tol2$ while there is about 60s between the two rotations (see Fig. 11). The fastest detection of all is shown in Fig. 10.

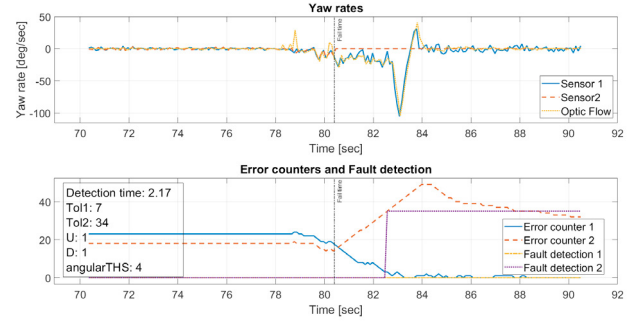


Fig. 10. Flight1 with total sensor failure.

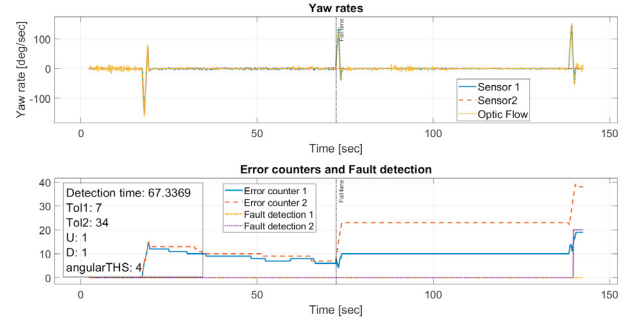


Fig. 11. Flight2 with total sensor failure.

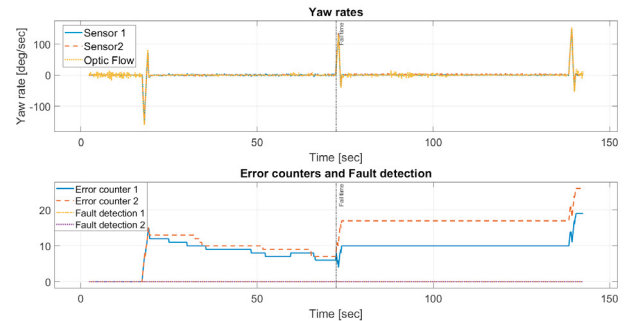


Fig. 12. Flight2 stuck with additive-type sensor failure.

5. CONCLUSION

The paper presents optical flow-based vertical angular rate fault detection considering real flight data. After introducing the optical flow calculation and the method for vertical angular rate estimation the two flight test trajectories are introduced. They were flown above an artificial urban area (ZalaZONE Smart City) with a gimbal stabilized downward looking camera. Presentation of the image processing method is followed by adding artificial sensor errors to onboard IMU measurements to simulate faults. Fault detection is based-on a residual threshold and up-down counters.

The parameters were tuned considering false alarm and missed detection rate and the receiver operating characteristics (ROC) curve. Analysis of the developed method shows that it is capable to detect most of the faults (additive above $2.1^\circ/s$ and multiplicative above 8%) but usually requires a long time for detection. This is rather caused by the lack of persistent excitation than by any flaw of the method. The only real flaw is the inability to detect small bias errors, or sensor stuck at a small value and this is planned to be solved by calculating the Euler angles from the angular rates and comparing the cumulated rotation with the image-based results.

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