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Cardiovascular Model Identification Using Neural ODE

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Abstract:

Acute circulatory failure (ACF) is a clinical syndrome when the heart and circulatory circulation cannot provide adequate blood supply to meet metabolic needs of the organs. ACF affects 30%–50% of intensive care unit (ICU) patients. Fluid resuscitation is the primary treatment of ACF. However, it fails in a significant proportion (about 50%) of cases due to lack of clinically feasible non-invasive perfusion markers to assess the efficacy of the fluid therapy. Unfortunately, unsuccessful fluid therapy negatively affects patient outcome, increasing ICU length of stay and costs. Recent studies show identifying Stressed Blood Volume (SBV) of the cardiovascular system can be used to assess the potential efficacy of fluid therapy. The development of the diagnostic method requires the identification of the central arterial pressure curve based on the femoral arterial pressure, which is clinically available. This central arterial pressure curve can be used to identify the cardiovascular system parameters. In this study, the main goal was to develop a parameter-identification method for the Tube-load model-based transfer function connecting the femoral and central arterial pressure curve by using the so-called Physics-informed Neural Network methodology, namely the Neural ODE method. The study presents the adaptation of the Neural ODE method to the given parameter identification problem and the validation of the developed identification method. The robustness of the developed identification method was tested and used on a series of measurement data recorded in animal experiments.

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Keywords: personalized model-based medical therapy methods, cardiovascular system, physiological system modelling, parameter identification, transfer function, Neural ODE, intensive care

1. INTRODUCTION

Fluid therapy is a widely used intervention to stabilize heart function in the case of Acute Circulatory Failure (ACF) (Messina et al. (2020); Monnet et al. (2023); Malbrain et al. (2024)). However, the effectiveness of fluid therapy is not directly predictable, and the therapy can also be harmful. Physiological models are able to predict fluid responsiveness, but require aortic (central arterial) blood pressure (Pironet et al. (2015); Murphy et al. (2019, 2020a,b)).

The measurement of central arterial blood pressure is highly invasive. Thus, it is not a commonly available

measurement in clinical practice. However, the aortic pressure can be approximated using femoral arterial pressure measurements (Murphy et al. (2021); Murphy and Chase (2023))). The Tube-load model (Zhang et al. (2011); Smith et al. (2021)) describes the relationship between the femoral and central arterial blood pressure if correct parameters are known.

This research aims to develop a parameter identification method for this transfer function using the so-called Neural ODE method which is known to be a robust approach for numerical identification problems (Chen et al. (2018)).

2. METHODS

2.1 Data

The experimental data set was collected from a series of animal experiments performed at the Centre Hospitalier Universitaire de Liège (Reference Number 14-1726). In the experiments, a cohort of 7 pure Piétrain pigs, weighing 18.5 kg to 29.0 kg, were used. The animals were initially sedated and anaesthetized. Sedation and anaesthesia were maintained via a continuous infusion of pharmaceutical drugs, namely sufentanil, Thiobarbital and Nimbex, delivered via a superior vena cava catheter. Pigs were mechanically ventilated. High-fidelity pressure catheters were used in the proximal aorta (P_{ao}), femoral artery (P_{fem}), and vena cava (P_{cv}) to measure blood pressure. Left ventricular pressures (P_{LV}) and volumes (V_{LV}) were also measured. Flow into the aorta (Q_{ao}) was measured using an aortic flow probe positioned on the proximal aorta near the aortic valve. Once the probe was located, the thorax was held closed using clamps. All data was measured with a sampling rate of 250 Hz and recorded as a single Notocord data file. Thus, all data required for model validation was recorded.

The pigs underwent different haemodynamic interventions during the measurements. During mechanical ventilation for every pig, a number of respiratory recruitment manoeuvres were performed in which positive end-expiratory pressure (PEEP) is increased in steps of 5 cmH₂O. Increasing positive end-expiratory pressure reduces systemic venous return to the right heart and increases pulmonary resistance. Intravenous infusions of saline solution were infused into the pigs several times and delivered at a rate of 500mL over 30 min. This intervention aims to increase circulatory volume and ventricular preload, thus SV. An infusion of endotoxin (*E. Coli* lipopolysaccharide) at a rate of 0.5mgkg⁻¹ was applied over 30 minutes to produce a septic shock-like response: inflammation, capillary leakage, decreased afterload, hypovolemia, tissue hypoxia and eventual cardiac failure. The schedule of these interventions is shown in Figure 1.

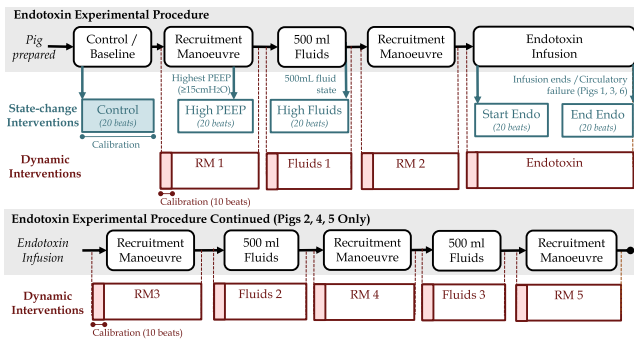


Fig. 1. The schedule of interventions performed on the examined pigs (Smith (2022))

2.2 Tube-load model

The Tube-load model describes the arterial system as an arbitrary number of parallel uniform lossless tubes, representing the large conduit arteries, in series with loads, representing peripheral blood vessels, as shown in Figure 2. (see also Zhang et al. (2011)).

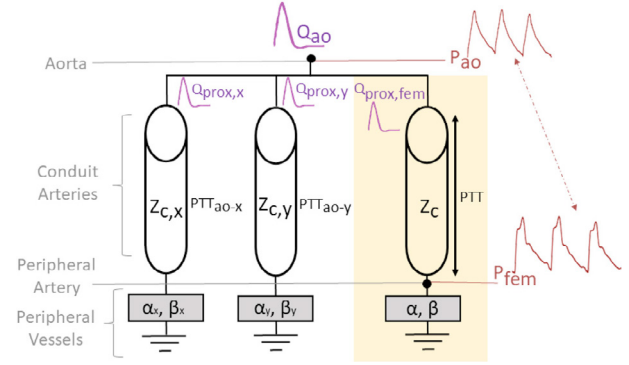


Fig. 2. The schematic figure of the Tube-load model (Smith et al. (2021))

The time it takes for pressure waves generated by the heart (the left ventricle) to travel through the large arteries to the periphery is called pulse transit time (PTT). These waves are also reflected, which leads to differences between Q_{ao} and P_{ao} waveform shapes, as the reflected wave increases pressure, but decreases flow. The Tube-load model captures this effect by modelling forward- and backward-travelling waves along the uniform tubes.

Each tube and load represents a pathway to a different peripheral vascular bed. In the model, the pressure of each tube at the proximal end is the aortic pressure (P_{ao}). The aortic flow (Q_{ao}) equals the sum of the proximal flows of all tubes. The femoral pressure (P_{fem}) is the distal pressure of a tube, and this parameter, together with PTT is used to estimate central flows and pressures ($Q_{ao,est}$, $P_{ao,est}$), assuming changes in the proximal femoral arteries are representative of the changes in the aorta. The characteristic impedance of the tube (Z_c), representing the resistance of the conduit arteries, is assumed constant. The impedance of distal arterial beds (Z_{load}) is modelled in the frequency domain as a single pole and zero, characterized by parameters A , B , with a gain factor Z_c :

$$Z_{load}(\omega) = Z_c \frac{j\omega + B}{j\omega + A}, \quad (1)$$

where ($0 < A < B$), and Z_c is impedance of the corresponding tube, ensuring pressure wave reflections are positive, stable, and become negligible at high frequencies.

As the frequency becomes very large, the peripheral load tends to Z_c , and the tube and load are impedance matched, so there are no reflections. The model can be used to create transfer functions between central and peripheral pressures and flows in the frequency domain. Thus, $P_{ao,est}$, and $Q_{ao,est}$ are calculated using measured P_{fem} and PTT (Smith et al. (2021); Swamy et al. (2008, 2009)):

$$\frac{P_{ao,est}(\omega)}{P_{fem}(\omega)} = \frac{\left(\frac{B+A}{2} + j\omega\right) e^{j\omega PTT} + \left(\frac{B-A}{2}\right) e^{-j\omega PTT}}{j\omega + B} \quad (2)$$

The transfer function can be represented in the continuous time domain:

$$\dot{P}_{ao,est}(t) + BP_{ao,est} = \dot{P}_{fem}(t + PTT) + \frac{B+A}{2} P_{fem}(t + PTT) + \frac{B-A}{2} P_{fem}(t - PTT) \quad (3)$$

Equation (3) describes the transfer function with parameters A, B and PTT , which we identify in this work.

Solving Equation (3) for the derivative of aortic pressure yields a differential equation enabling calculation of the aortic pressure based on an initial value and femoral pressure data.

$$\begin{aligned} \dot{P}_{ao,est}(t) = & -BP_{ao,est} + \dot{P}_{fem}(t + PTT) + \\ & + \frac{B+A}{2}P_{fem}(t + PTT) + \frac{B-A}{2}P_{fem}(t - PTT) \end{aligned} \quad (4)$$

This research aims to develop a Neural ODE-based parameter identification method to identify A and B in Equation (4). The transfer function takes the femoral arterial pressure wave as an input and outputs the aortic pressure wave's derivative.

2.3 Neural ODE

Neural Ordinary Differential Equations (Neural ODEs) are a new family of deep neural networks combining the fields of differential equations with artificial neural networks (Chen et al. (2018)). The combination of the two fields seems natural since both differential equations and neural networks aim to specify an arbitrary nonlinear transform. In the case of neural networks, this transform is defined using linear weights and a non-linear activation function for each layer. Ordinary differential equations define a non-linear transform by defining the derivative of the examined variable and thus leverage a priori knowledge of the structure of the problem.

As machine learning models grow and are hungry for larger and larger amounts of data, differential equations have become an attractive option for specifying nonlinearities in a learnable (via the parameters), but constrained form (Rackauckas et al. (2019)). Neural Ordinary Differential Equations are one of the ways ODEs and ANNs can be combined.

A starting point is the Residual Neural Network (ResNet). ResNets are neural networks characterized by skip connections with identity mappings, as seen in Figure 3.

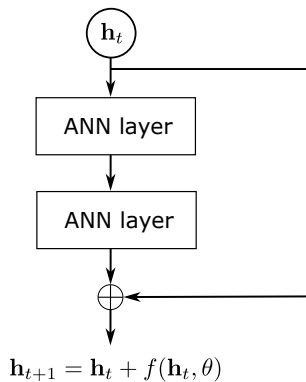


Fig. 3. Model of the residual neural network

The output is the sum of the layer output and input/hidden state:

$$\mathbf{h}_{t+1} = \mathbf{h}_t + f(\mathbf{h}_t, \theta) \quad (5)$$

where $t \in 0 \dots T$, $\mathbf{h}_t$ is the state variable vector of the t^{th} layer of the neural network ($\mathbf{h}_t \in \mathbb{R}^D$) and θ represents the weights of the network.

Equation (5) can be seen as an Euler discretisation of a continuous transformation. If so, we can produce the original differential equation by adding more layers to the ResNet and taking smaller steps. In the limit, the neural network can be defined by an ODE:

$$\frac{d\mathbf{h}(t)}{dt} = f(\mathbf{h}(t), t, \theta) \quad (6)$$

Starting from the input layer $\mathbf{h}(0)$, we can define the output layer $\mathbf{h}(T)$ to be the solution to this ODE initial value problem at some time T . This value can be computed by a black-box differential equation solver, which evaluates the hidden unit dynamics $f(\cdot)$ where necessary to determine the solution with the desired accuracy. Figure 4 contrasts these two approaches (Chen et al. (2018)).

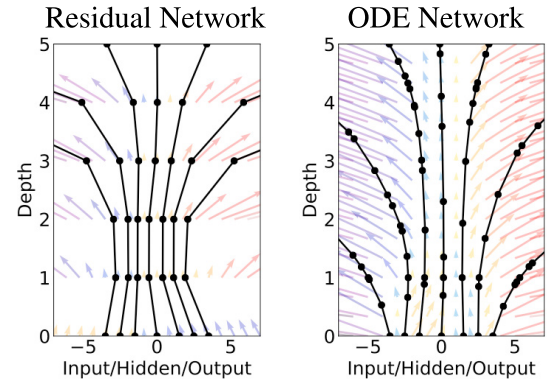


Fig. 4. The difference and similarity between ResNets and Neural ODEs (Chen et al. (2018)).

The main difficulty in training continuous-depth networks is performing backpropagation, which is the method to calculate the derivative of the loss function with respect to each parameter. The authors of the Neural ODE method treat the ODE solver as a black box and compute gradients using the adjoint sensitivity method, which is applicable to all ODE solvers. The adjoint sensitivity method calculates gradients by solving a second, augmented ODE backwards in time. The approach scales linearly with problem size, has low memory cost, and explicitly controls numerical error (Chen et al. (2018)).

2.4 Robustness assessment method

A benchmark test was designed to assess the robustness of the Neural ODE-based identification method regarding the initial parameter selection of the ODE.

The parameter identification is implemented by the training process of the neural network of Neural ODE model. In the neural network training iterations, in each epoch, the error between the measured aortic pressure and the solution of the ODE (Equation (4)) with the actual parameter set is calculated, and the network weights are modified to decrease the error. Thus, by the end of the iterations, the error is minimised, and, at the same time, the ODE parameters are identified. However, if the parameter identification method is imperfect, the solution of the ODE

will not fit the measurements, and the error will remain large even after training.

In the benchmark test designed, simulated aortic pressure curves, created by the solution of the ODE with one specific parameter set, were used instead of the measurements to examine the effects of only the identification method. Using the results of earlier studies, a common physiological parameter set is selected for the benchmark experiments: $A_{sim} = 4.6$, $B_{sim} = 16.0$, $PTT = 0.092$. The benchmark data set is created using these parameters in the solution of the differential equation of Equation (4) and using as the input the first, normal physiological section of the experimental pigs data (Pig #7). Figure 5 shows a section of the simulated data.

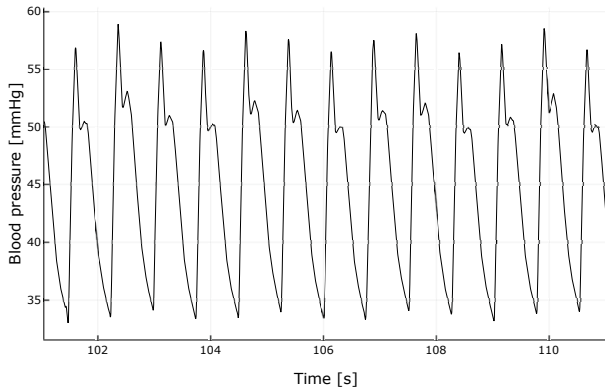


Fig. 5. Section of the simulated central aortic blood pressure signal used for the benchmark test

3. RESULTS

3.1 Estimating the robustness of the used method

During the benchmark test, the learning process was started from different initial parameters, semi-randomly sampled from the parameter space. The tested parameter domain covers all possible physiologically feasible parameter combinations and their wide environment: $A_{sim} - 65 < A < A_{sim} + 65$, $B_{sim} - 35 < B < B_{sim} + 65$, where A_{sim} and B_{sim} are the parameters used to create the benchmark data set. From parameters close to A_{sim} and B_{sim} , values were sampled with higher density, and from farther, with lower density. Figure 6 shows the sampling of the parameter space. The red cross indicates the A and B used for simulation (A_{sim} and B_{sim}), and the blue dots indicate the different initialization points used during the benchmark test.

During the simulation, the trajectory of the parameters could be visualized starting from different initialization points during the different epochs of learning. The parameters during learning are shown in Figure 7. Each subfigure shows the current locations in the parameter space A and B at specific iteration steps, such as 4000, 10000, 20000, and 40000, during the training process.

Figure 8 shows the calculated mean squared error for a given heart cycle in a wide range of the A and B parameters. On this heatmap, the global minimum where the parameters should converge is clearly visible.

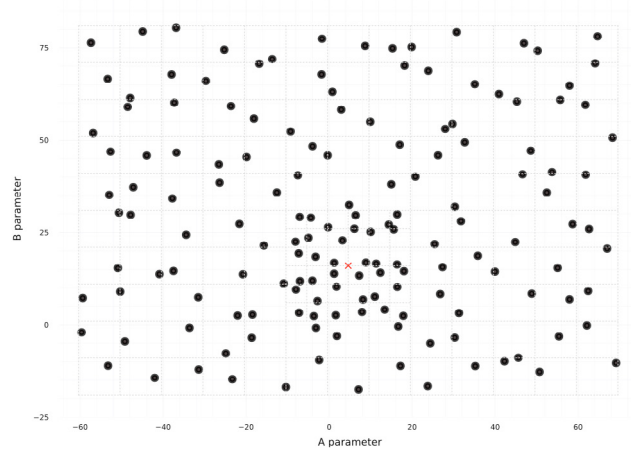


Fig. 6. The sampling of the parameter space, red cross indicates ($A_{sim}; B_{sim}$), and the blue dots indicate the different initialization points used during the benchmark test.

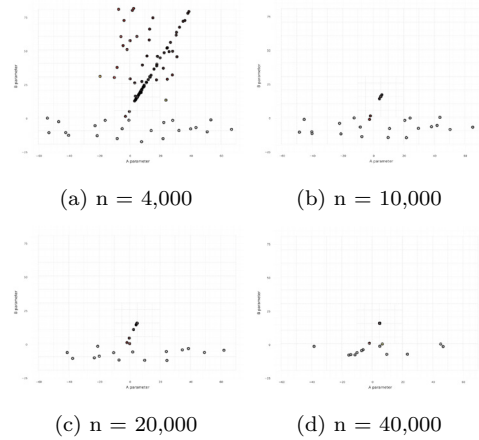


Fig. 7. The resulting parameters values after $n * 1000$ iterations in case of $n=4$ (a), $n=10$ (b), $n=20$ (c) and $n=40$ (d).

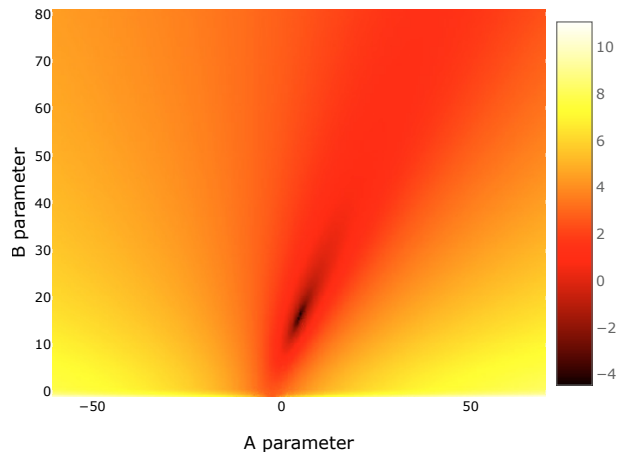


Fig. 8. Heatmap: logarithm of the mean squared error with different simulation parameters for a given heart cycle

3.2 Applying the identification method to real measurements

After the estimation of the robustness of the identification method, the method is applied to real measurements.

Two sections from one experimental pig data (Fig7) were selected for the first evaluation. The first section is at the very beginning of the measurements. This section is cut from the beginning of the data set, shortly after the catheters have been placed, and can be considered as a base or normal section. During this section, no interventions were started; the pig was in normal physiological conditions. This section of data is shown in Figure 9.

During the second selected part, fluid (a saline solution of 500mL) was infused into the pig. The infusion of the fluid changes haemodynamics: increases circulatory volume and ventricular preload, and, as a result, increases stroke volume. The second section is shown in Figure 11.

A subset of both sections is visualized in Figures 10 and 12 for better interpretability.

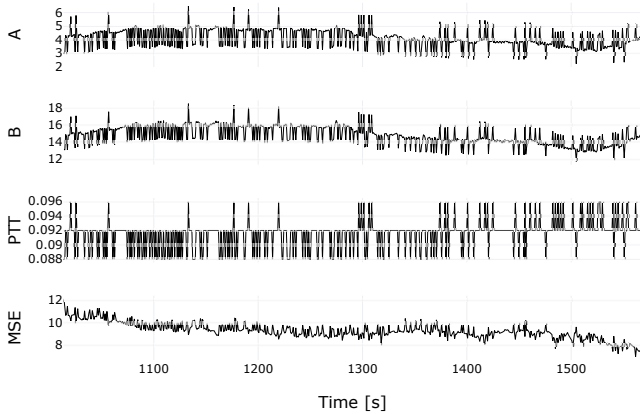


Fig. 9. Identified parameters of the first selected section before any interventions were made. This section can be considered as a normal physiological section without any changes in the haemodynamics. A and B parameters were identified with the Neural ODE method, while PTT was calculated from the measurement. In the bottom plot, the loss function can be seen.

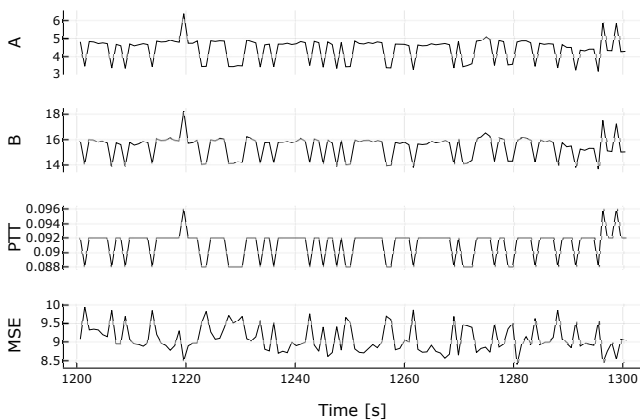


Fig. 10. Enlarged section of the trajectories shown in Figure 9, where the identified A and B parameters can be seen better.

4. DISCUSSION

Looking at the plots (Figure 7a-7d), there is an obvious correlation between the A and B parameters, as the majority of the points are gradually moving to a line

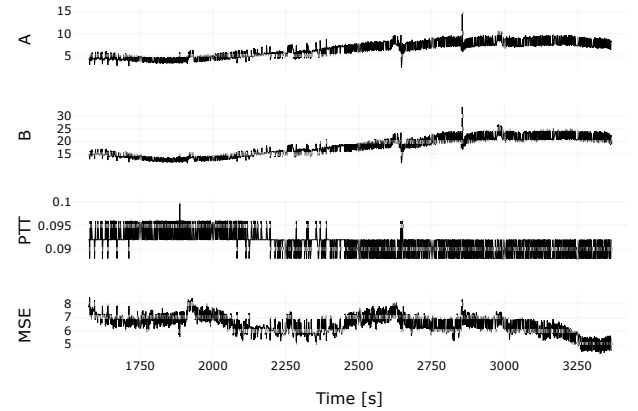


Fig. 11. Identified parameters of the second selected section where a saline solution of 500mL was infused into the pig. During this section, there is a change in haemodynamics that should result in increased stroke volume. A and B parameters were identified with the Neural ODE method, while PTT was calculated from the measurement. In the bottom plot, the loss function can be seen.

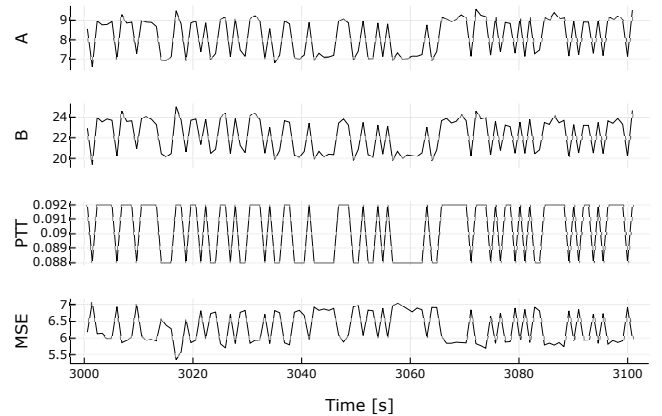


Fig. 12. Enlarged section of the trajectories shown in Figure 11, where the identified A and B parameters can be seen better.

during the learning process. The linear correlation is also seen on the heatmap in Figure 8. Figure 8 clearly shows there is a global minimum in the parameter AB space. In Figures 7a-7d, the different initial points are clearly converging to the global minimum. However, it is likely there are local minima or a very low slope plateau of the loss function, as there are some points on the bottom side of the tested parameter set where A parameter is below 0, which do not or very slowly converge to the simulated parameters. However, most of the points do converge to the simulation parameters and only < 5% of points do not converge. Also, the theoretical minimum value of A is 0 according to the physical constraints of the Tube load model. Thus, if the initialization is appropriate, which means the initial parameter values are close enough to the actual parameters and are in a physically valid domain, then the method is robust, and the parameters converge to the optimum.

In the second part of the evaluation, the identification method was applied to real measurements. In the first,

normal physiological section, the A and B parameters (Figure 9) remain quite stable. In the second section (Figure 11), fluid was infused into the pig that changed haemodynamics — accordingly, A and B increased.

This study proved the applicability of the Neural ODE-based identification method for the identification of the A and B parameters of the transfer function derived from the Tubeload model. Using the available experimental data set and the derived benchmark data, the method was found to be robust. The applied experimental data set is designed to cover a relatively wide range of physiological states. However, the validation was limited to only one pig experiment. Thus, this study was not able to test the effect of variability between different animals which will be the subject of a subsequent study.

5. CONCLUSION

The research aimed to examine the applicability of the Neural ODE method for the parameter identification of the transfer function describing the relationship between the central arterial pressure curve and the femoral arterial pressure curve derived from the Tube-load model. As shown, the Neural ODE method was successfully adapted for the identification problem. The robustness of the Neural ODE-based identification method regarding the initial parameter selection of the ODE was assessed using a benchmark test. Based on the benchmark test results, the method is robust if the parameters are initialized according to the physiologically valid parameter domain.

The developed identification framework can be used to analyse larger data sets and identify the transfer function parameters for different physiological conditions. Thus, the outcomes of this research can lead to the definition of the transfer function parameter set domain belonging to different normal and pathological haemodynamic situations.

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REFERENCES

- Chen, T.Q., Rubanova, Y., Bettencourt, J., and Duvenaud, D. (2018). Neural ordinary differential equations. *CoRR*, abs/1806.07366. URL <http://arxiv.org/abs/1806.07366>.
- Malbrain, M.L., Wong, A., Nasa, P., and Ghosh, S. (2024). Rational use of intravenous fluids in critically ill patients.
- Messina, A., Collino, F., and Cecconi, M. (2020). Fluid administration for acute circulatory dysfunction using basic monitoring. *Ann Transl Med*, 8(12), 788.
- Monnet, X., Malbrain, M.L., and Pinsky, M.R. (2023). The prediction of fluid responsiveness. *Intensive Care Medicine*, 49(1), 83–86.
- Murphy, L., Chase, J.G., Davidson, S.M., Smith, R., and Desaive, T. (2020a). Minimally invasive model based stressed blood volume as an index of fluid responsiveness. *IFAC-PapersOnLine*, 53(2), 16257–16262. doi:10.1016/j.ifacol.2020.12.621. 21st IFAC World Congress.
- Murphy, L., Chase, G., Davidson, S., and Desaive, T. (2021). Estimating central blood pressure from a single peripheral pressure measurement. *IFAC-PapersOnLine*, 54(15), 532–537. 11th IFAC Symposium on Biological and Medical Systems BMS 2021.
- Murphy, L. and Chase, J.G. (2023). Single measurement estimation of central blood pressure using an arterial transfer function. *Computer Methods and Programs in Biomedicine*, 229, 107254.
- Murphy, L., Davidson, S., Chase, J.G., Knopp, J.L., Zhou, T., and Desaive, T. (2020b). Patient-specific monitoring and trend analysis of model-based markers of fluid responsiveness in sepsis: A proof-of-concept animal study. *Annals of biomedical engineering*, 48(2), 682–694.
- Murphy, L., Davidson, S., Knopp, J.L., Chase, J.G., Zhou, T., and Desaive, T. (2019). State analysis of total stressed blood volume and arterial elastance during induced sepsis. In *2019 41st Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC)*, 2951–2954.
- Pironet, A., Dauby, P.C., Chase, J.G., Kamoi, S., Janssen, N., Morimont, P., Lambermont, B., and Desaive, T. (2015). Model-based stressed blood volume is an index of fluid responsiveness. *IFAC-PapersOnLine*, 48(20), 291–296. 9th IFAC Symposium on Biological and Medical Systems BMS 2015.
- Rackauckas, C., Innes, M., Ma, Y., Bettencourt, J., White, L., and Dixit, V. (2019). DiffEqFlux.jl - A julia library for neural differential equations. *CoRR*, abs/1902.02376. URL <https://arxiv.org/abs/1902.02376>.
- Smith, R., Murphy, L., Pretty, C.G., Desaive, T., Shaw, G.M., and Chase, J.G. (2021). Tube-load model: A clinically applicable pulse contour analysis method for estimation of cardiac stroke volume. *Computer Methods and Programs in Biomedicine*, 204, 106062. doi: <https://doi.org/10.1016/j.cmpb.2021.106062>.
- Smith, R.G.R. (2022). *Model-based hemodynamic management of critically ill patients*. Ph.D. thesis, University of Canterbury.
- Swamy, G., Mukkamala, R., and Olivier, N.B. (2008). Estimation of the aortic pressure waveform from a peripheral artery pressure waveform via an adaptive transfer function. In *2008 30th Annual International Conference of the IEEE Engineering in Medicine and Biology Society*, 1385–1388. doi:10.1109/IEMBS.2008.4649423.
- Swamy, G., Xu, D., Olivier, N.B., and Mukkamala, R. (2009). An adaptive transfer function for deriving the aortic pressure waveform from a peripheral artery pressure waveform. *American Journal of Physiology-Heart and Circulatory Physiology*, 297(5), H1956–H1963. doi: 10.1152/ajpheart.00155.2009.
- Zhang, G., Hahn, J.O., and Mukkamala, R. (2011). Tube-load model parameter estimation for monitoring arterial hemodynamics. *Frontiers in physiology*, 2, 72.